Assimilation of monitoring data to improve slope reliability

WP4: Monitoring Data Analysis and Modelling

Muhammad Mohsan, Femke C. Vossepoel, Philip J. Vardon

Department of Geoscience and Engineering, Delft University of Technology, Netherlands

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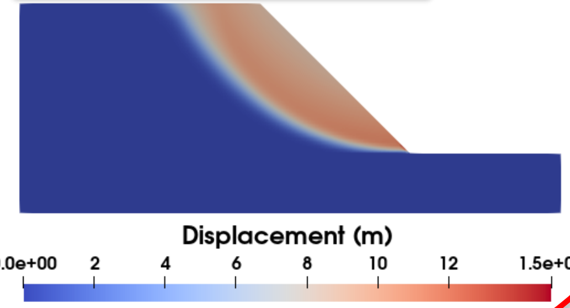
Introduction

Numerical model

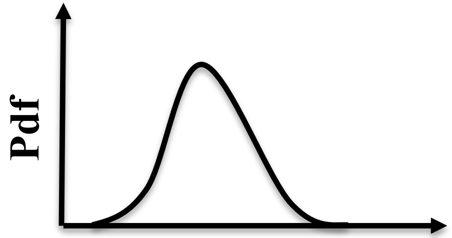


- Numerical approximations
- Missing physics
- Poorly known parameters

- Estimate state
- Estimate parameters
- FoS estimation

Data assimilation combines the model and observations to find the reliable outcome

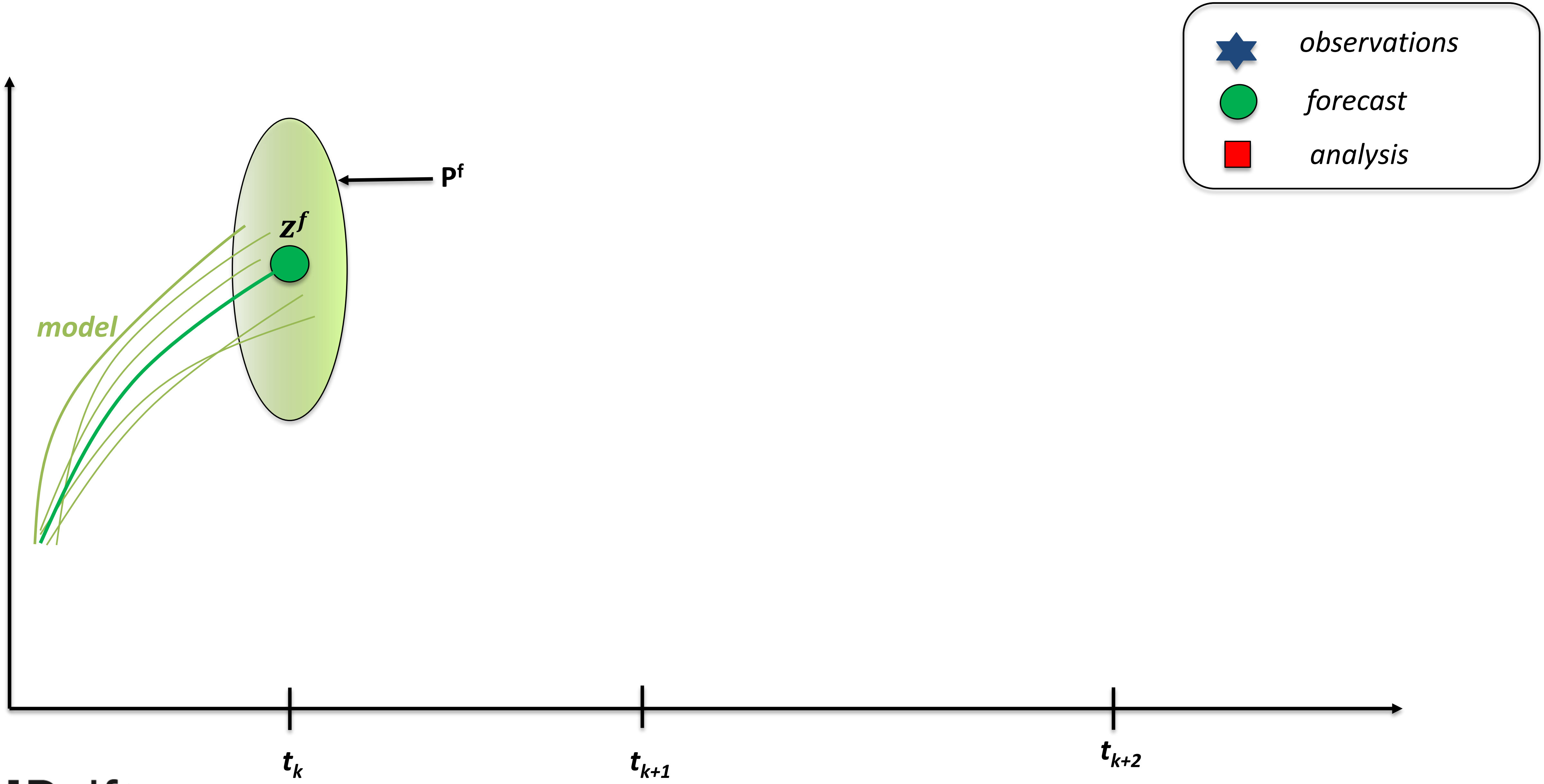
Pdf of FoS



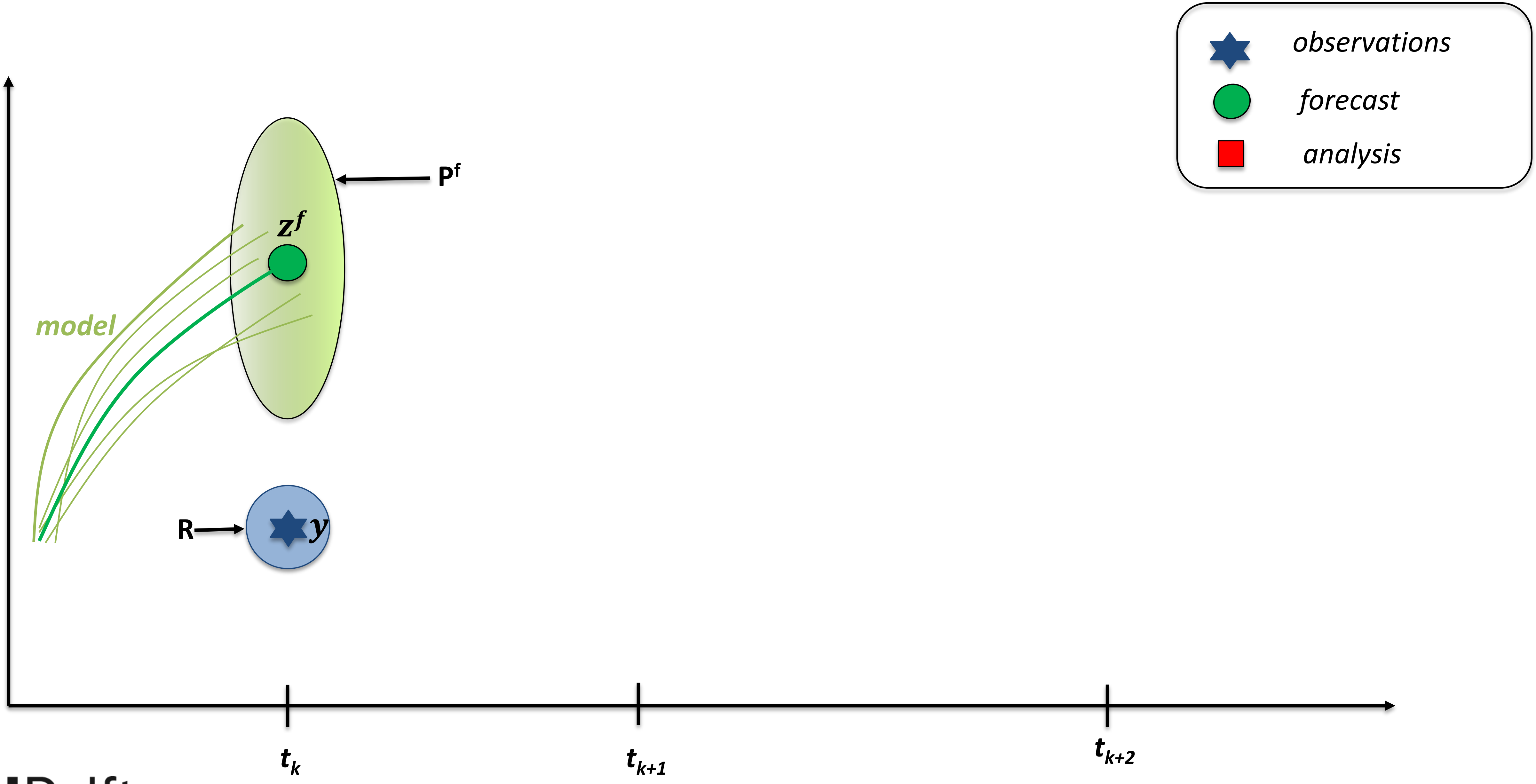
Observations

- Observation errors

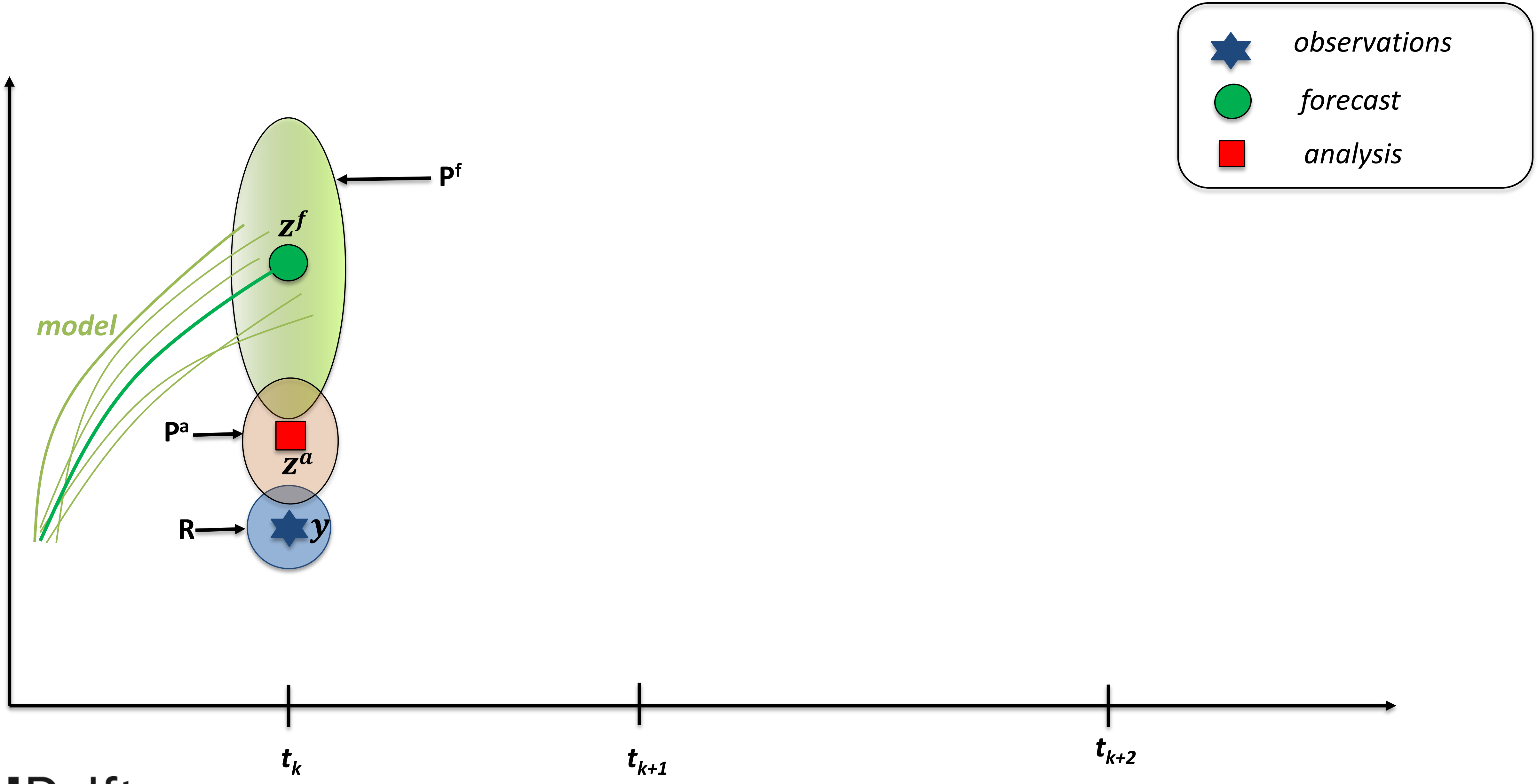
Data Assimilation



Data Assimilation



Data Assimilation



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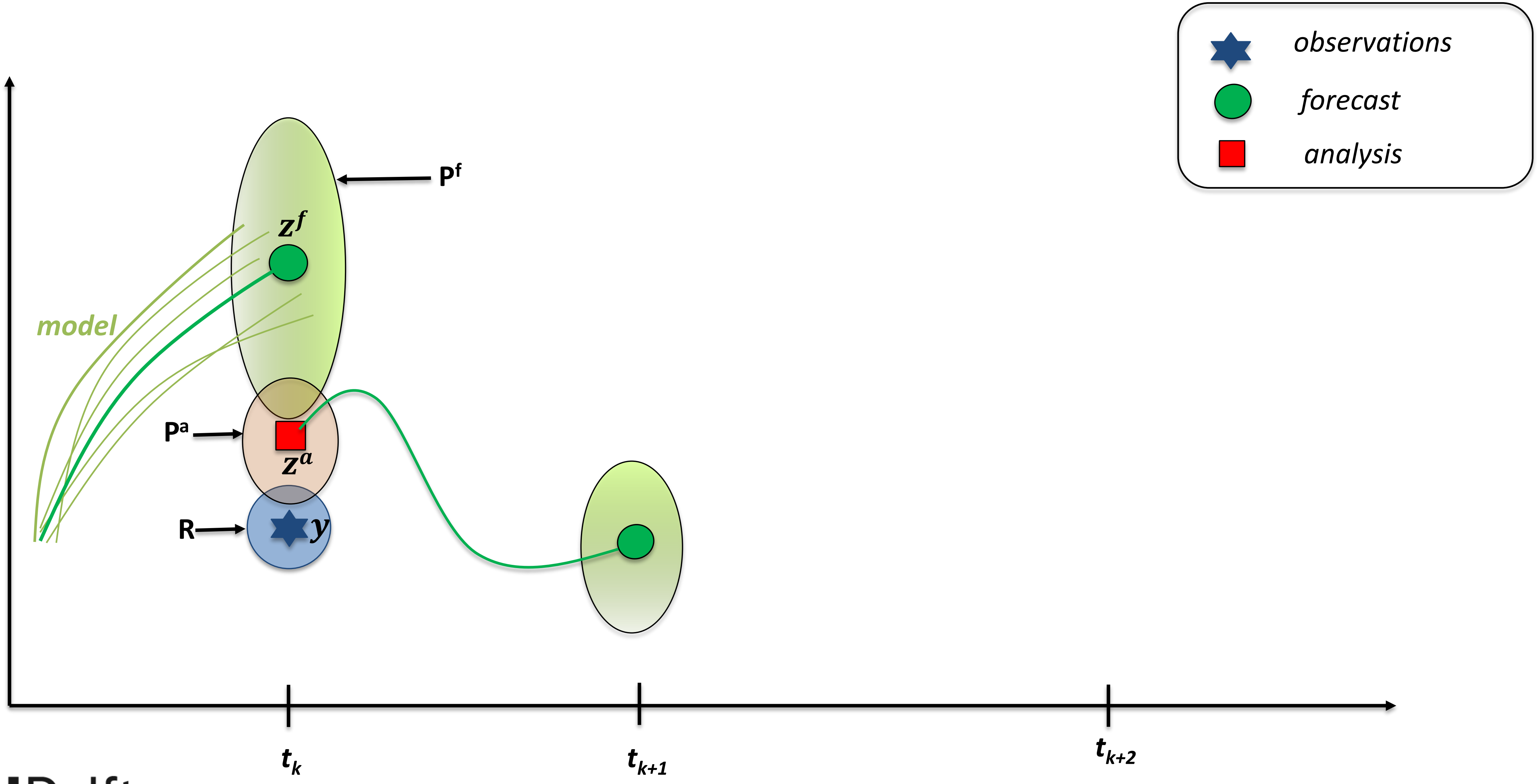
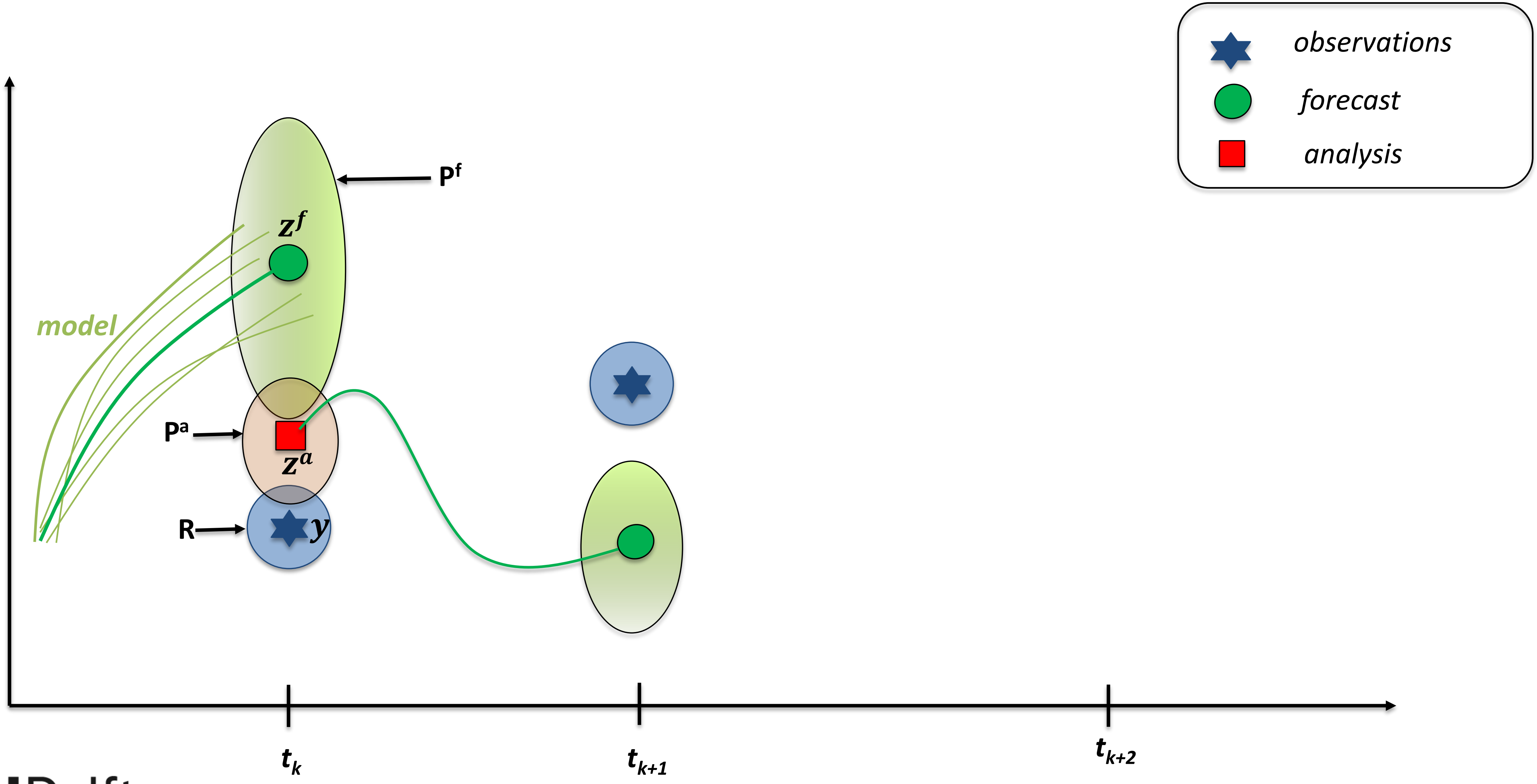
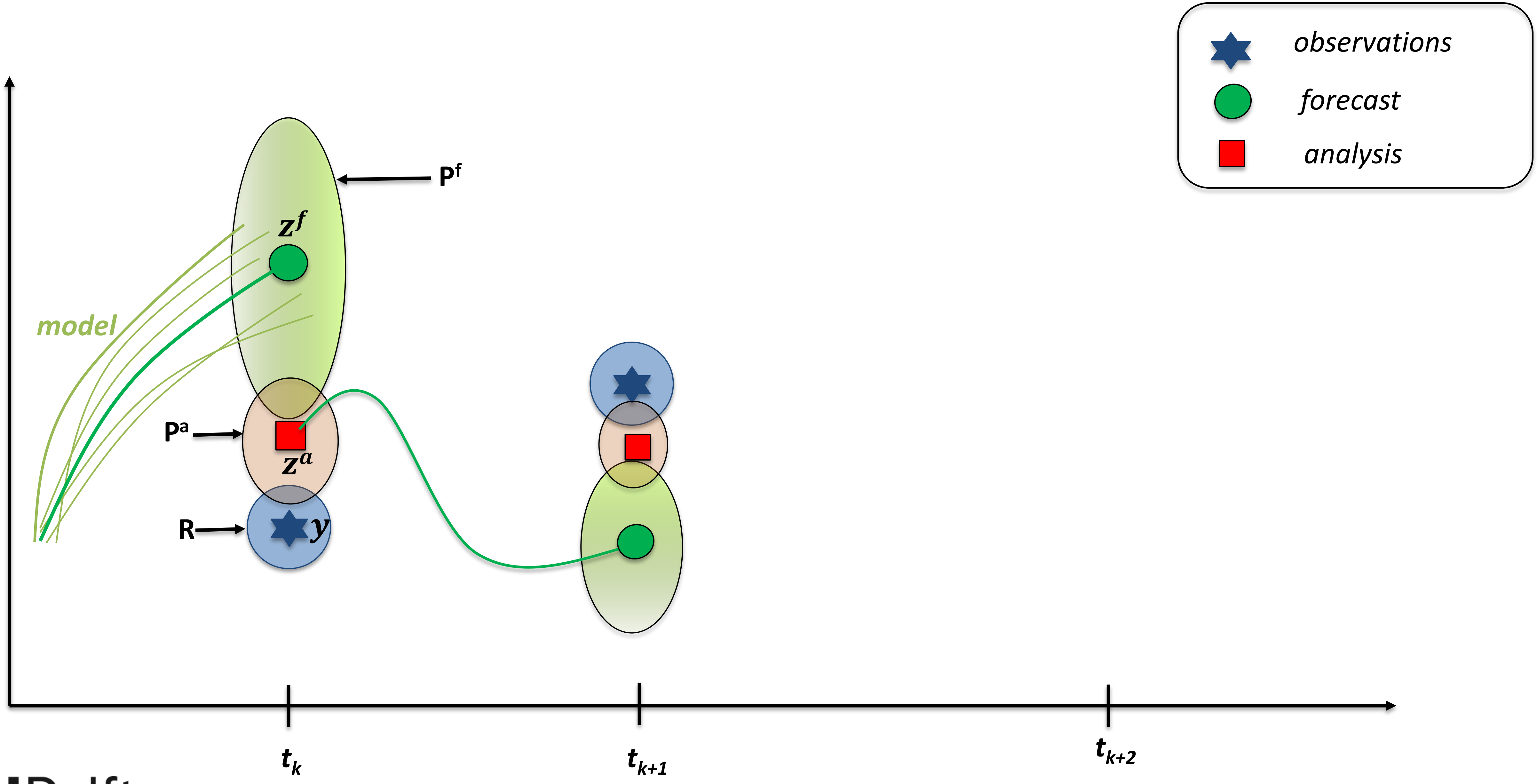
Data Assimilation



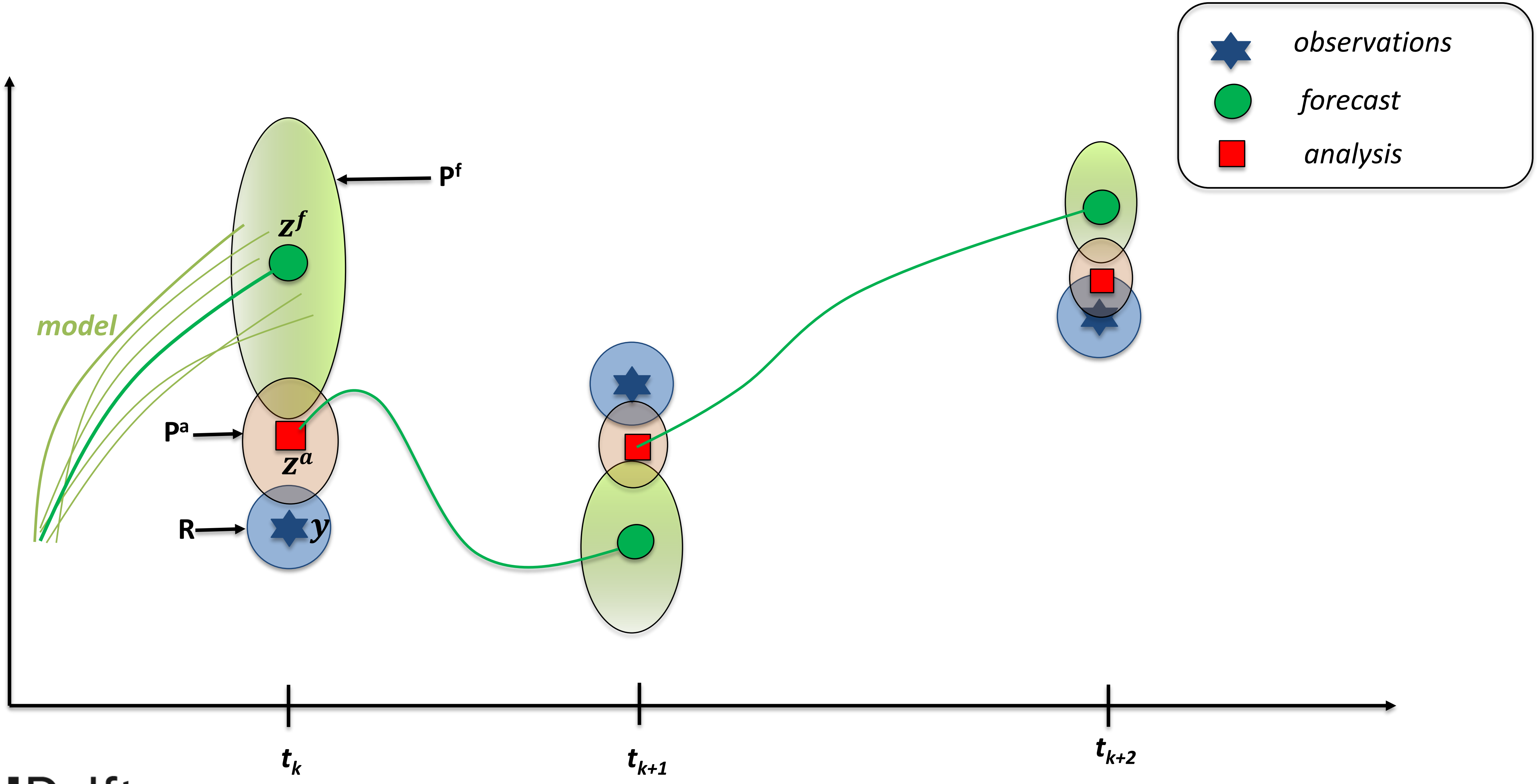
Data Assimilation



Data Assimilation



Data Assimilation



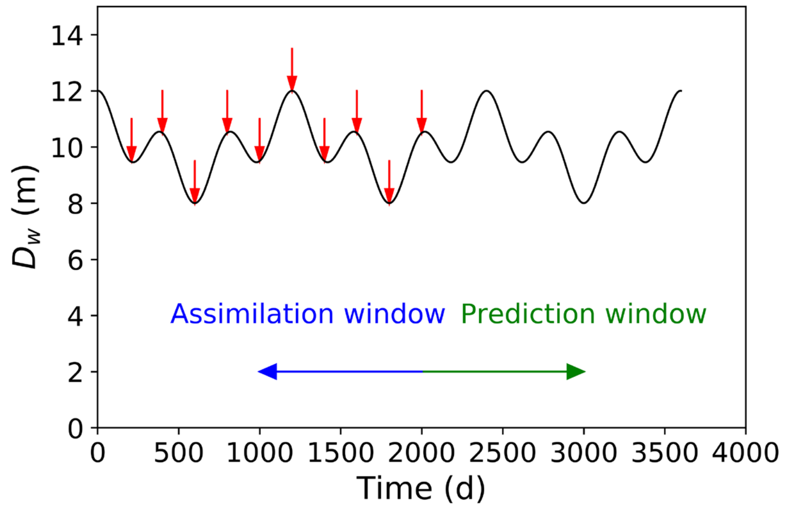
Modified from Tandeo et al., 2020

To apply data assimilation in a slope stability system

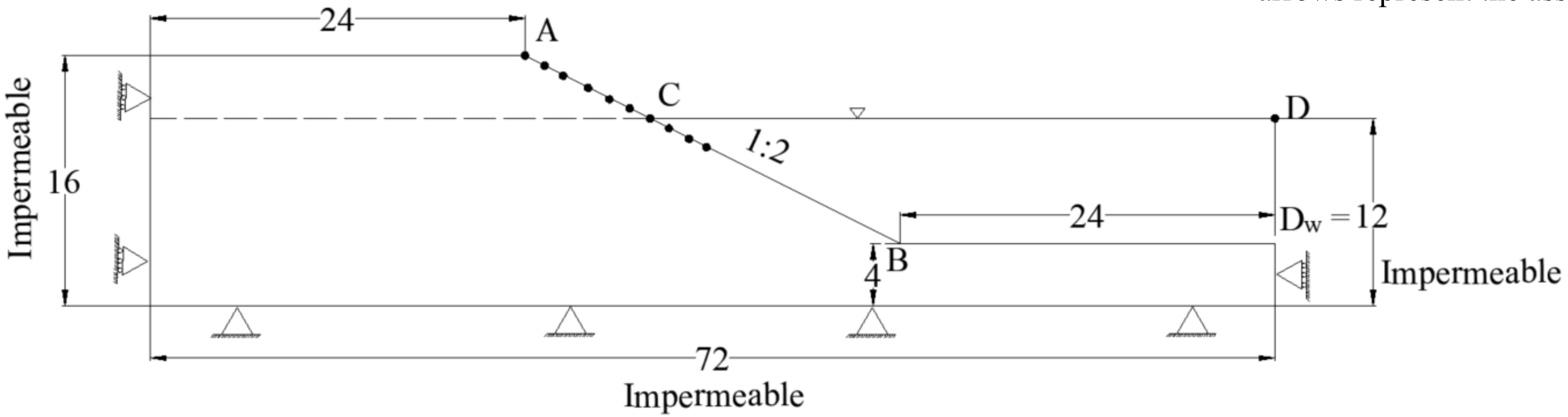
1. What sort of constitutive models we should use?
2. What sort of data assimilation techniques we should use?

Case 1:What sort of constitutive models we should use?

1. Numerical model : Finite element method
   a. Mohr-Coulomb (MC) model
   b. Hardening soil (HS) model
2. Data assimilation scheme : (Recursive) ensemble Kalman filter
   a. To estimate the strength and stiffness parameters based on <u>synthetic observations of slope deformation</u>.



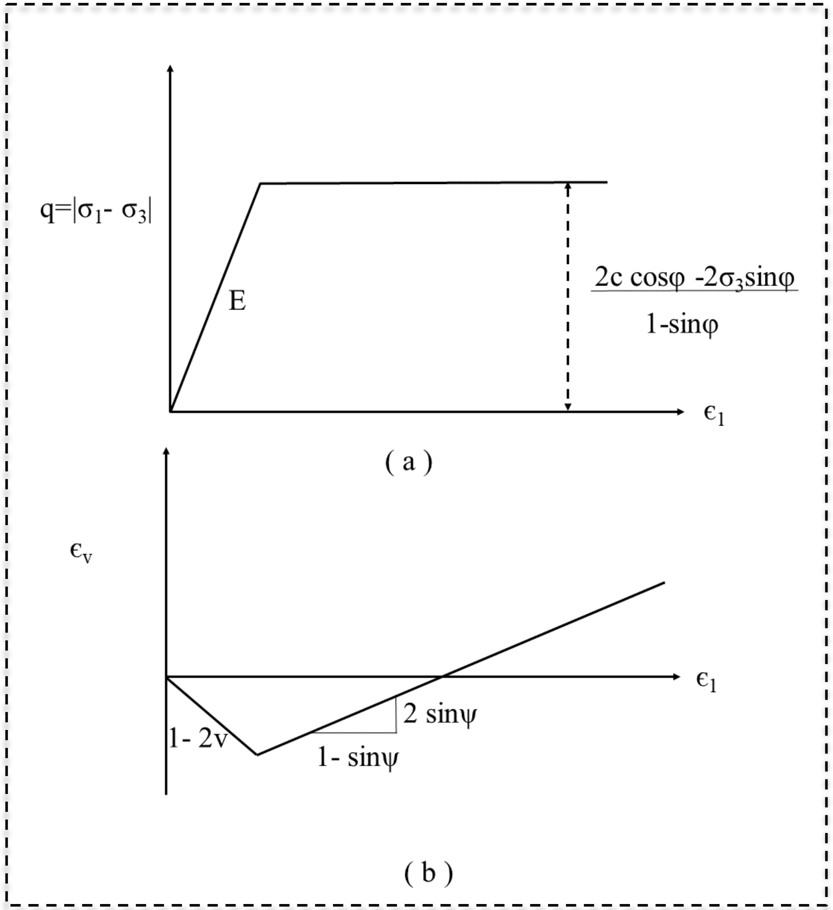
Fluctuation of the water level (Dw) in CD. The red arrows represent the assimilation times.



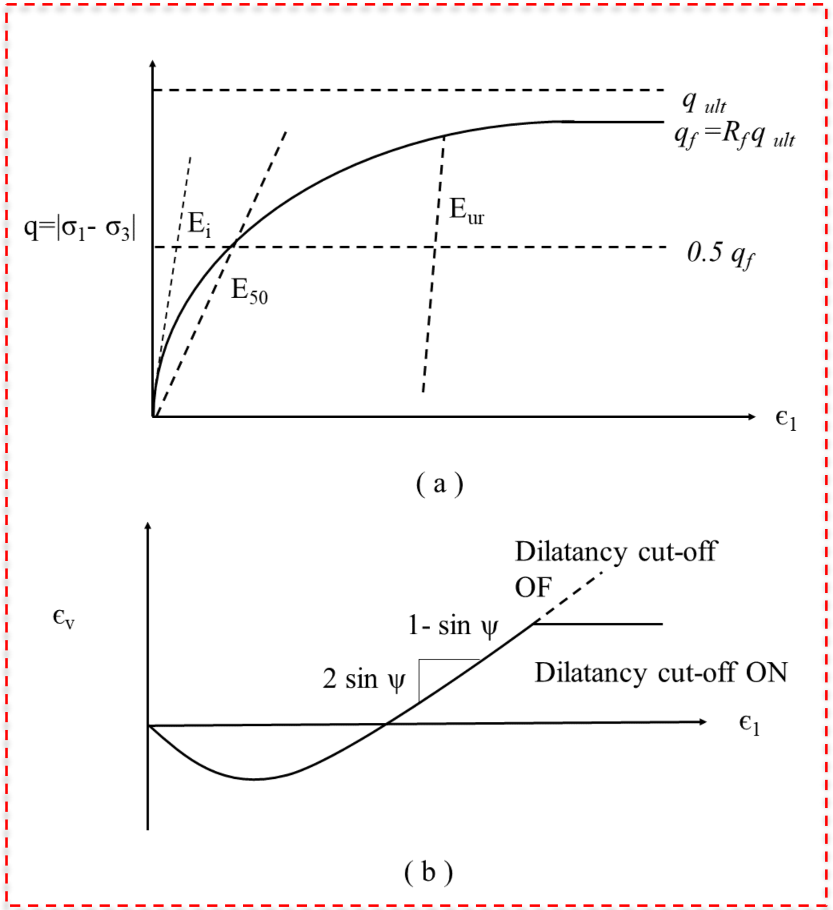
Geometry of the slope (dimensions in m) and black circles represents the measurement points.

Numerical model: FEM: Constitutive models

A set of mathematical equations which represents stress-strain behaviour in a given material is called the constitutive model.



Mohr-Coulomb (MC) model



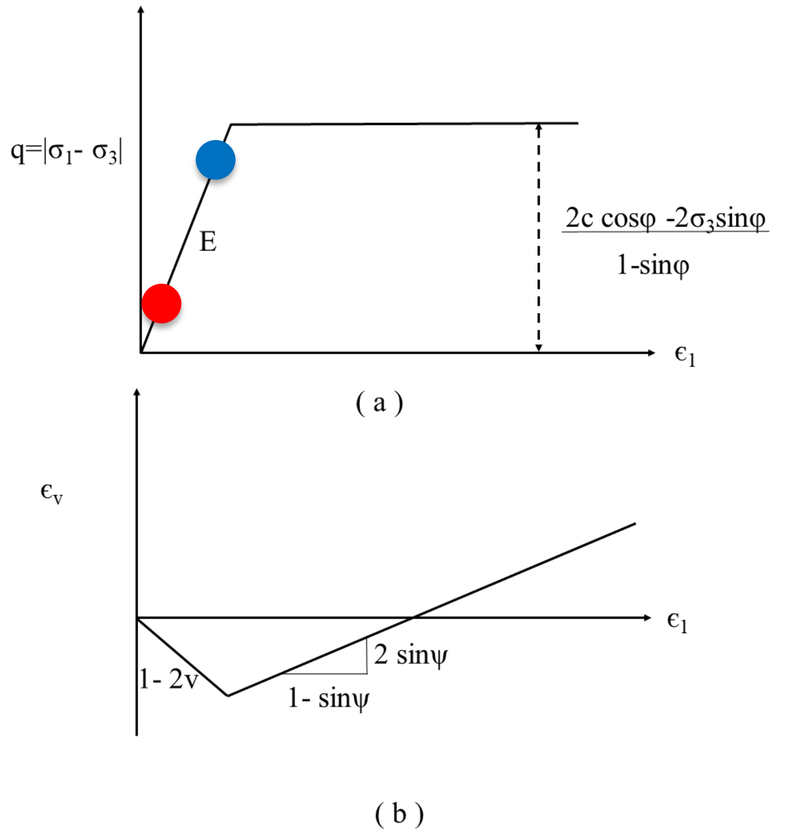
Hardening soil (HS) model

HS model

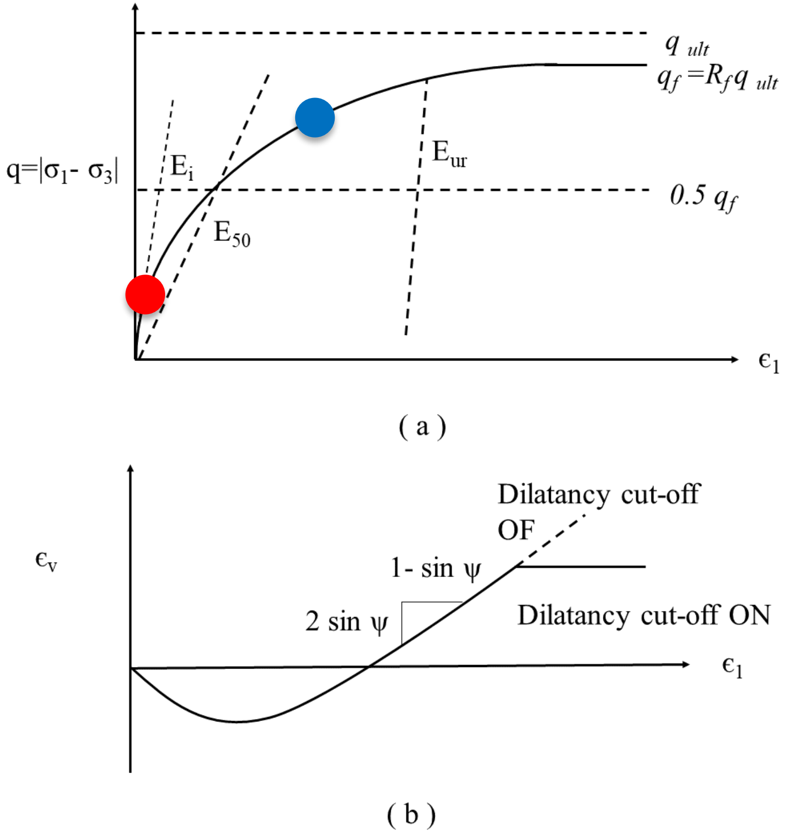
$$E_{50} = f(c,\ \varphi,\ \sigma_3)$$

Numerical model: FEM: Constitutive models

A set of mathematical equations which represents stress-strain behaviour in a given material is called the constitutive model.



Mohr-Coulomb (MC) model

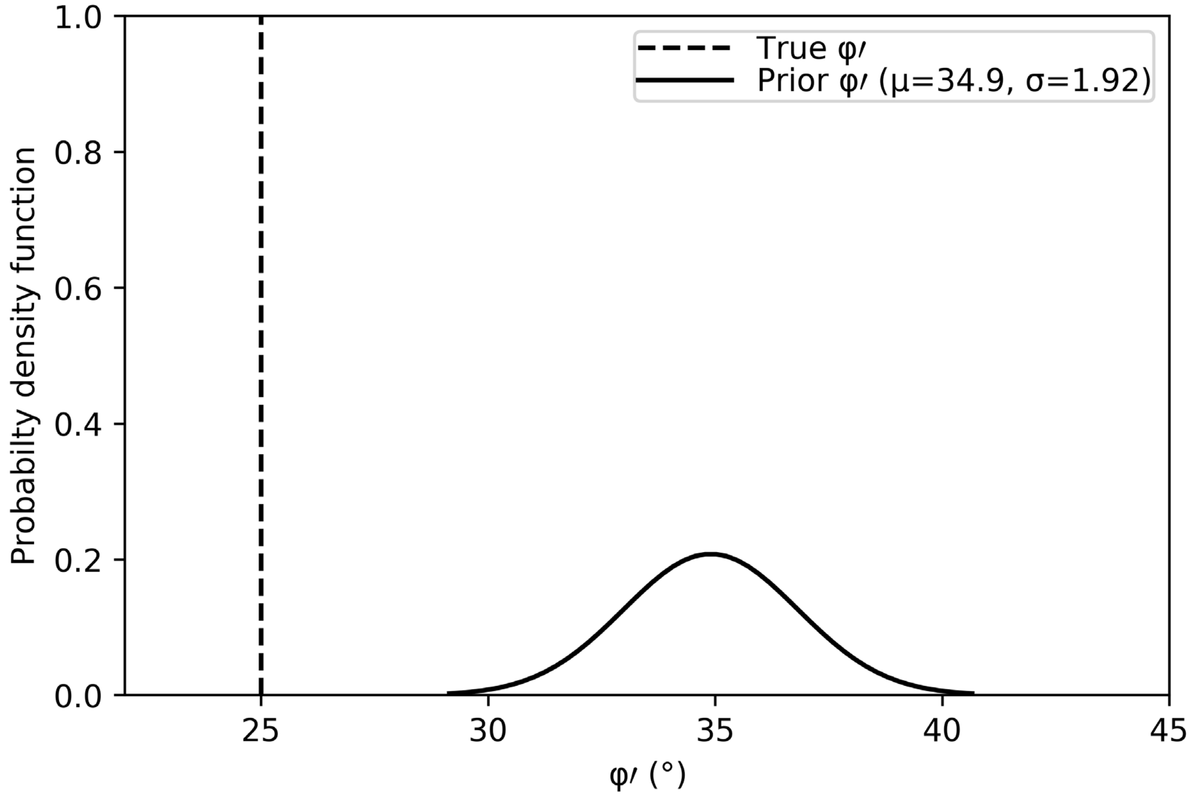


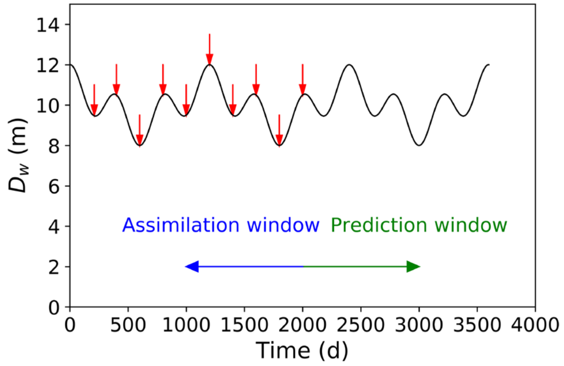
Hardening soil (HS) model

- Close-to-failure
- Far-from-failure

Parameter estimation

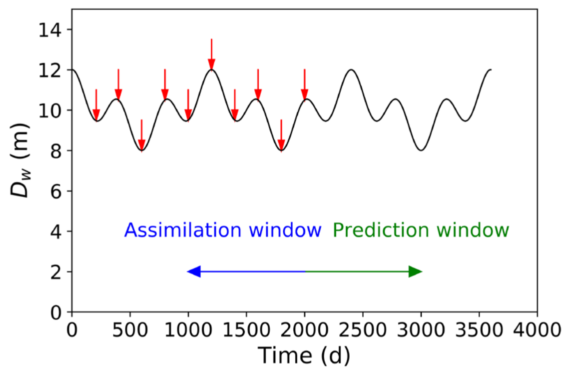
-Estimation of friction angle (φ') based on the slope deformation observations

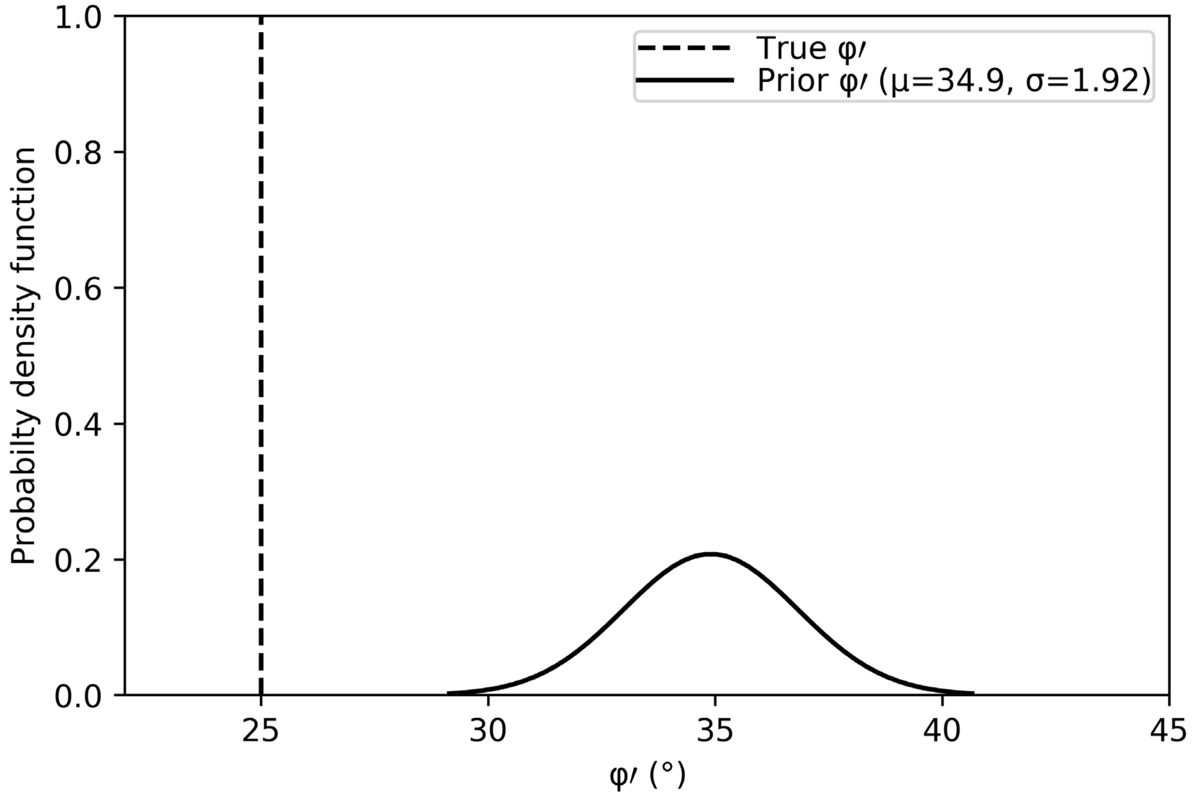




Estimation of friction angle for the MC and HS soil models at t=1000 and t=2000 days

Parameter estimation





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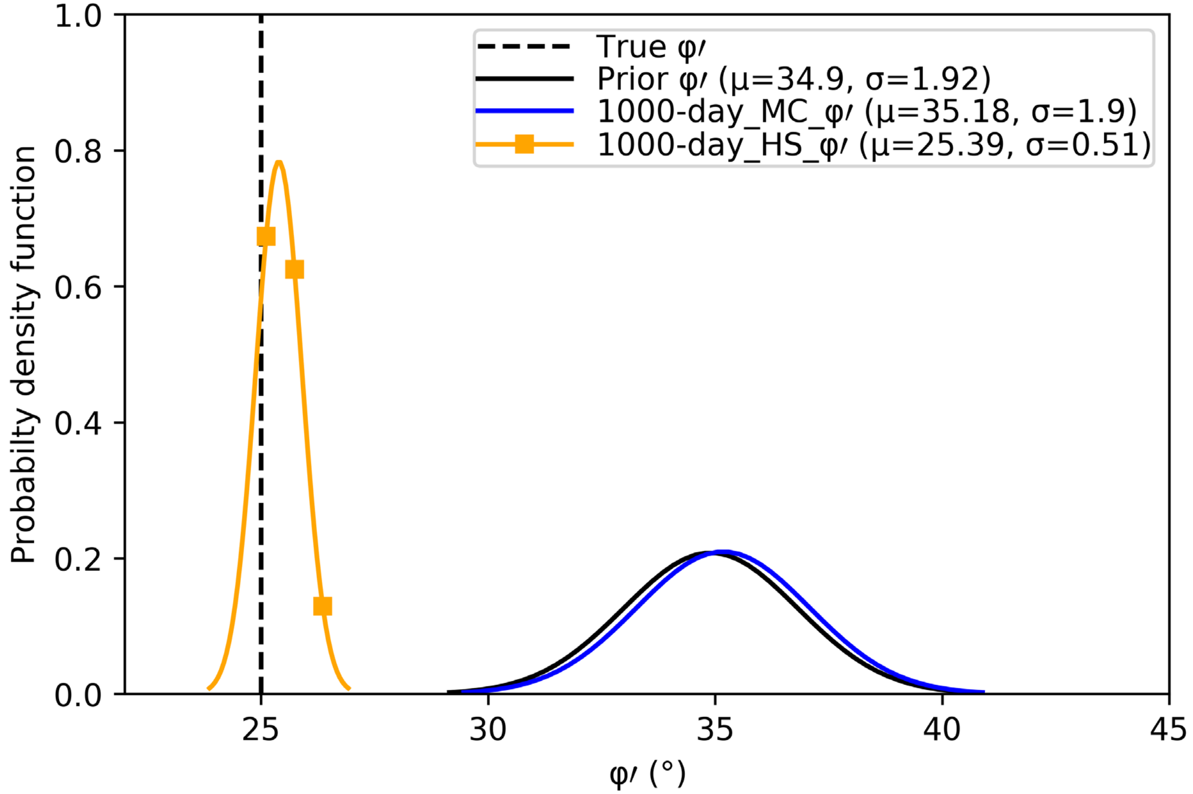

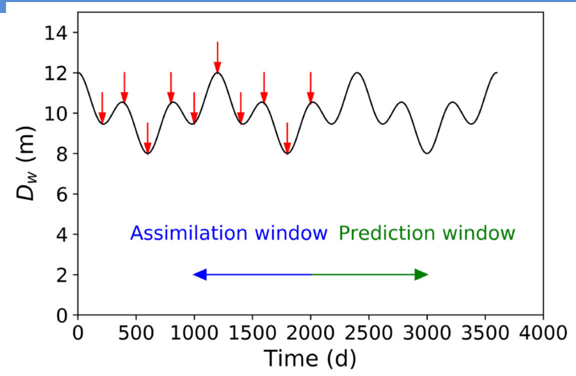

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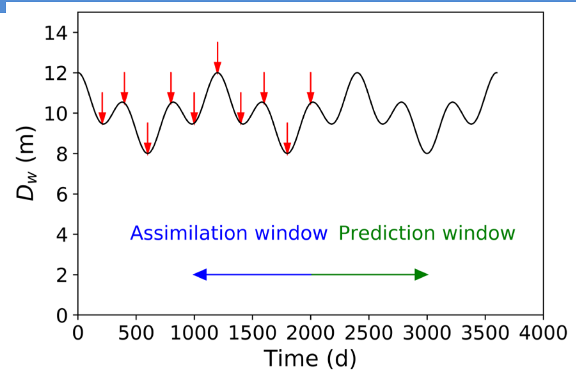
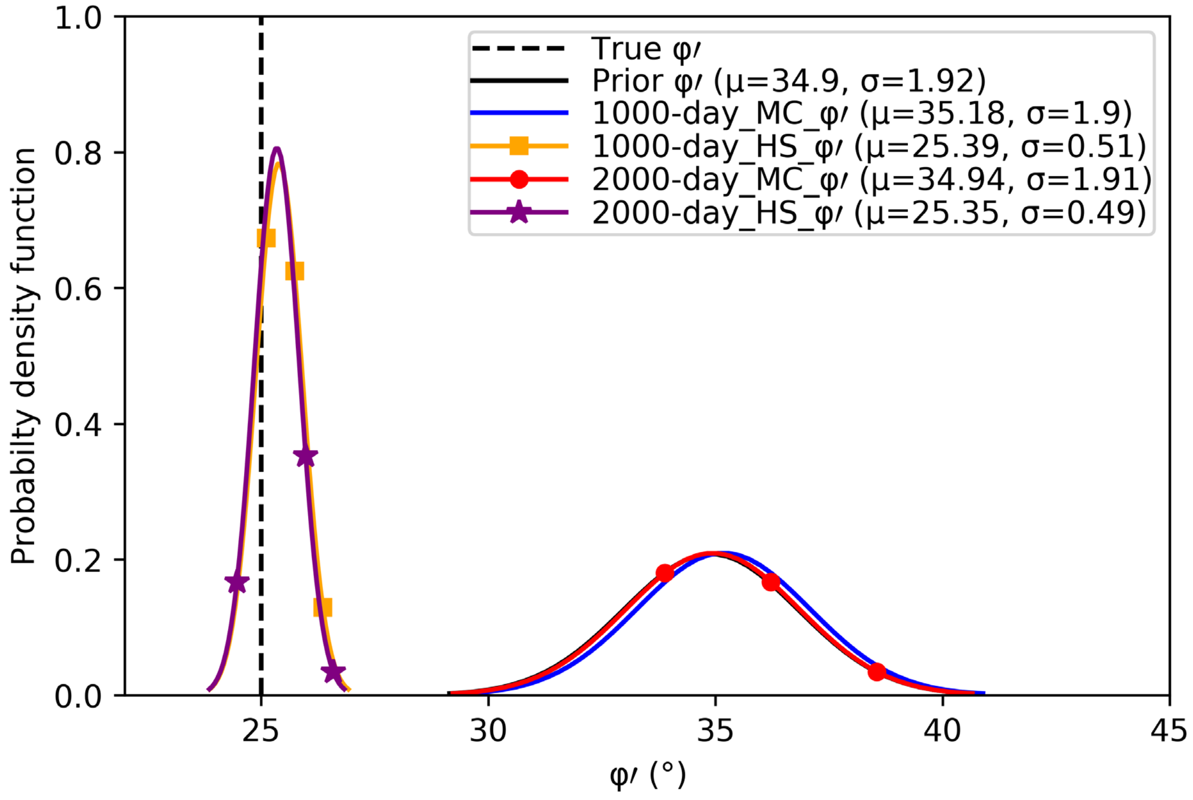
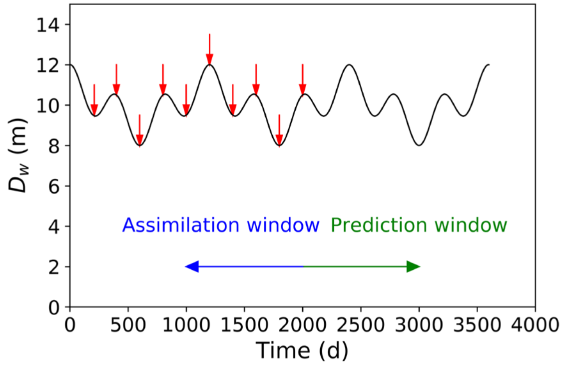
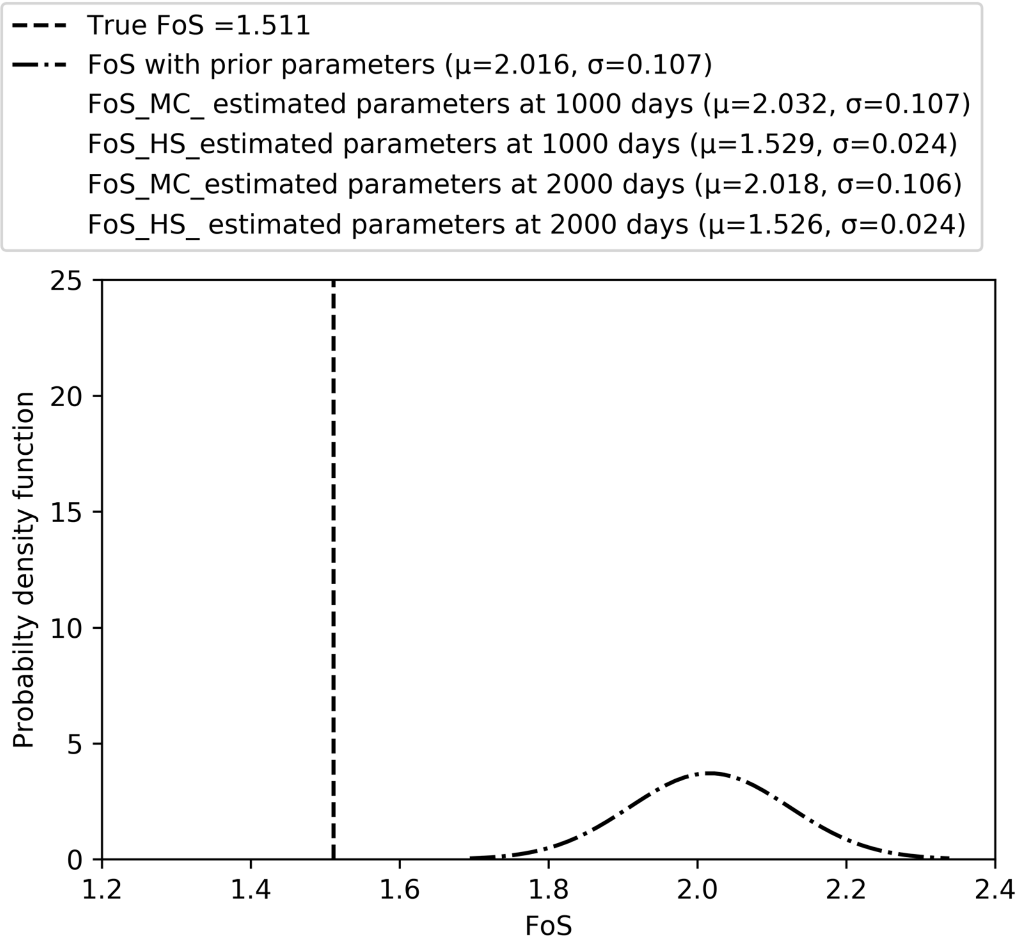




Estimation of friction angle for the MC and HS soil models at t=1000 and t=2000 days

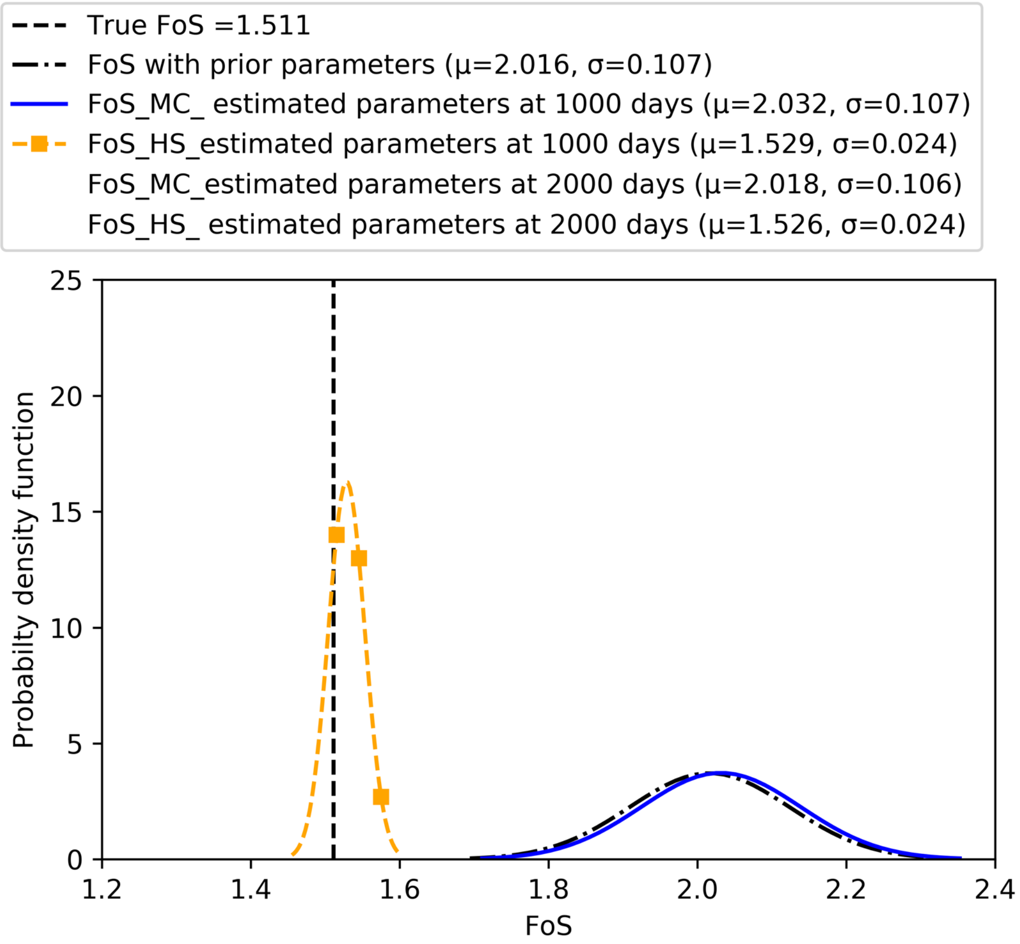
Factor of safety (FoS) estimation

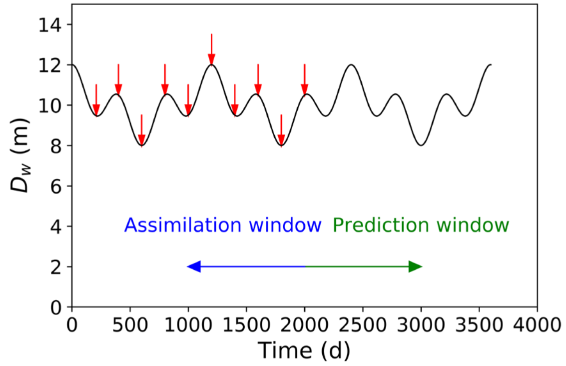




Probability distribution of factor of safety at 2000 days based on the prior and estimated parameters at 1000 and 2000 days

Factor of safety (FoS) estimation





Probability distribution of factor of safety at 2000 days based on the prior and estimated parameters at 1000 and 2000 days

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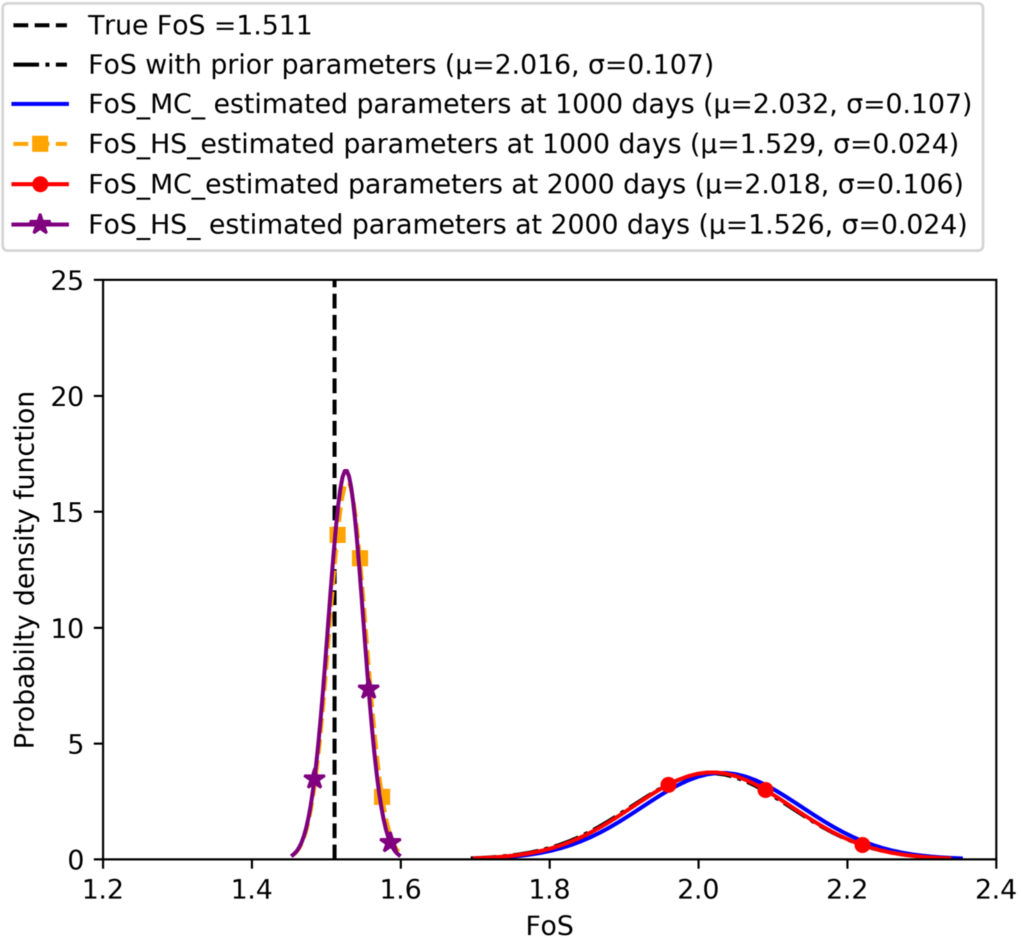

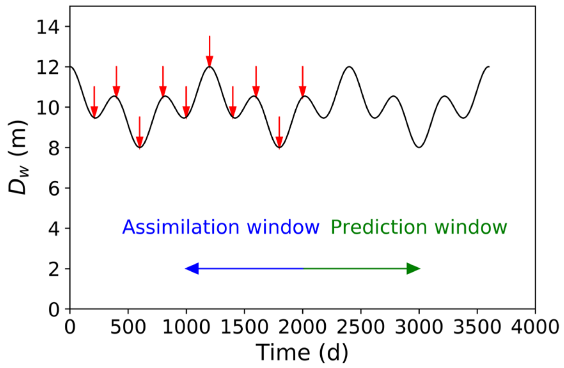

Probability distribution of factor of safety at 2000 days based on the prior and estimated parameters at 1000 and 2000 days

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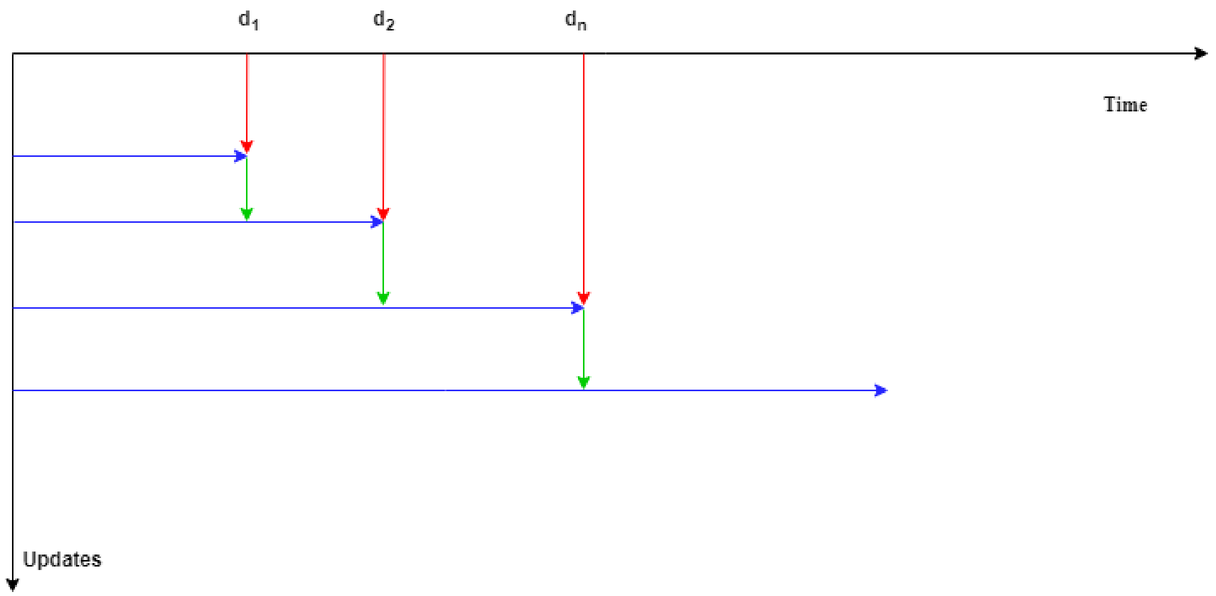
More details can be found on:

Mohsan, M., Vardon, P. J., & Vossepoel, F. C. (2021). On the use of different constitutive models in data assimilation for slope stability. Computers and Geotechnics, 138, [104332]. https://doi.org/10.1016/j.compgeo.2021.104332

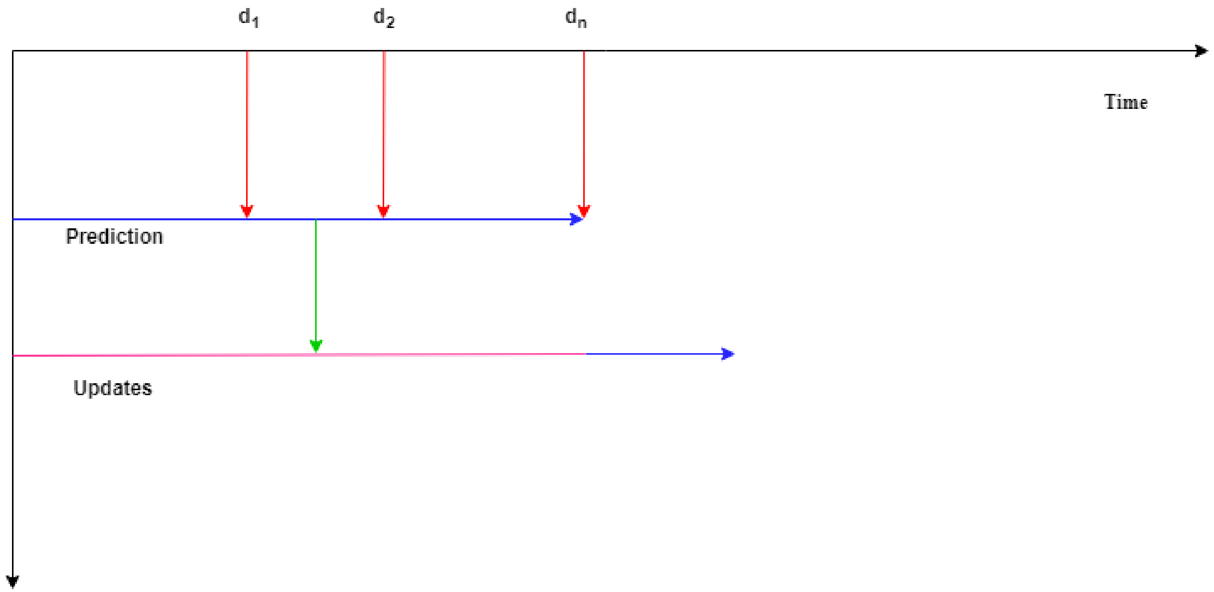
Case 2: What sort of data assimilation we should use?

1. Numerical model : Finite element method
    a. ~~Mohr-Coulomb (MC) model~~
    b. Hardening soil (HS) model
2. Data assimilation scheme :
    a. (Recursive) ensemble Kalman filter
    b. Ensemble smoother

Data assimilation techniques



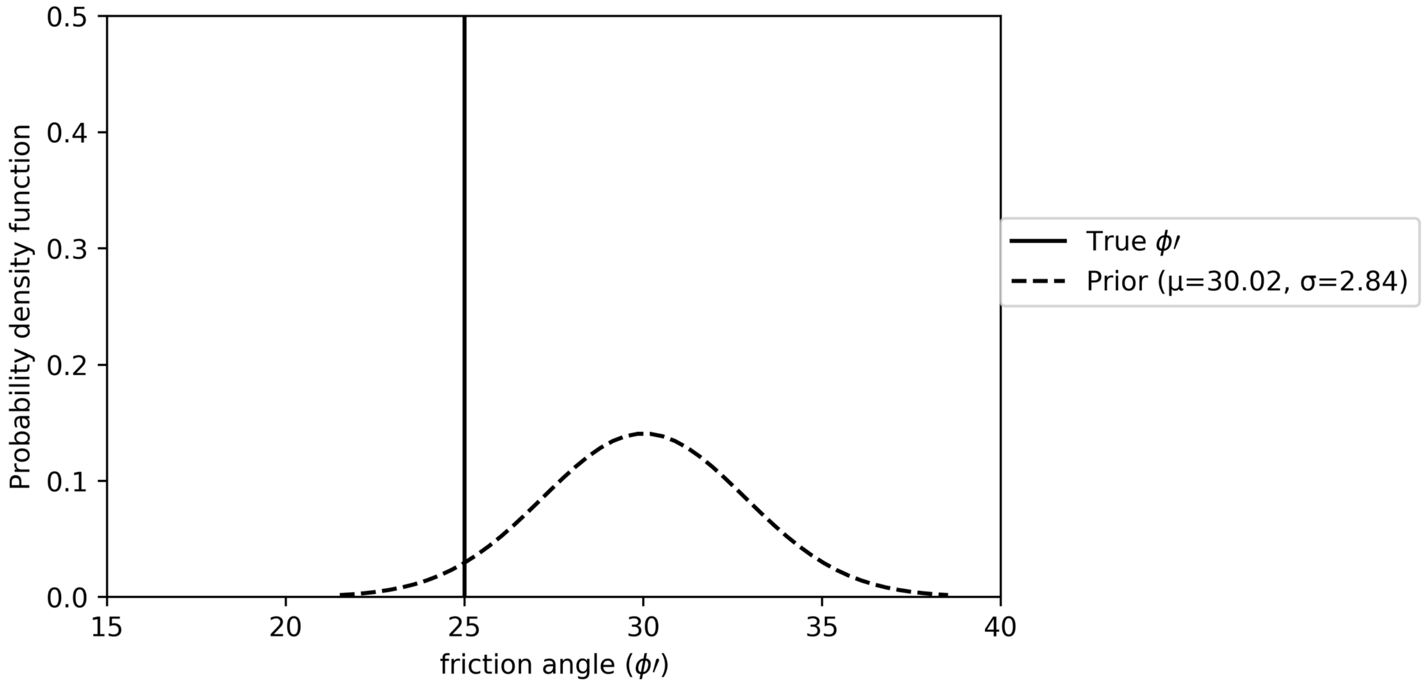
Ensemble Kalman filter (EnKF)



Ensemble smoother (ES)

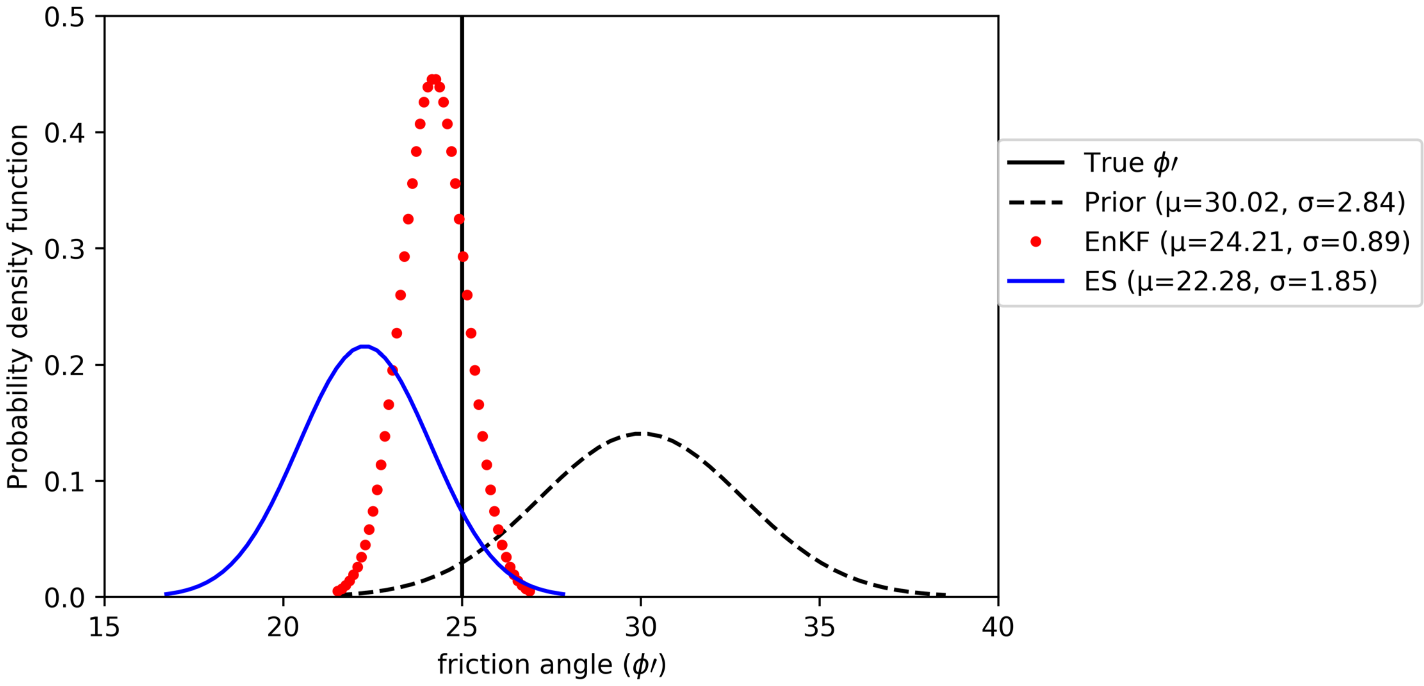
Parameter estimation

-Estimation of friction angle (φ') based on the slope deformation observations



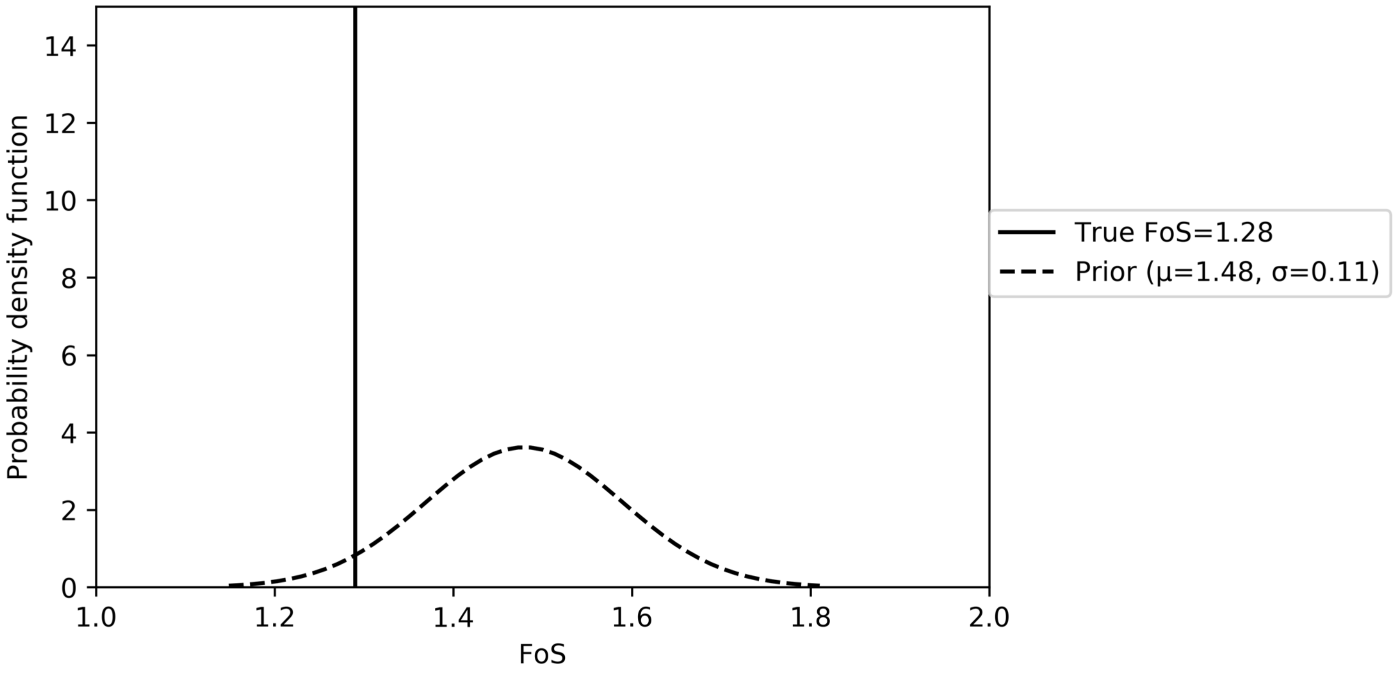
Estimation of friction angle by EnKF and ES at 6th time step

Parameter estimation



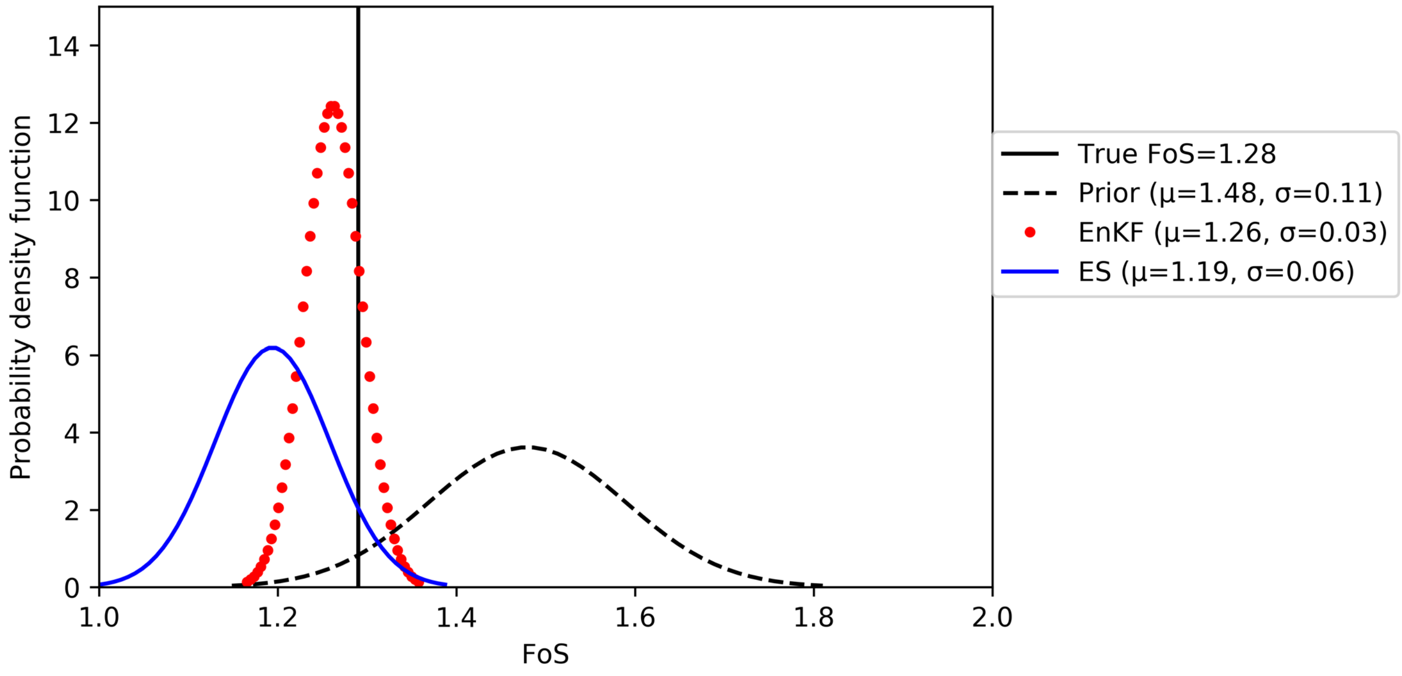
Estimation of friction angle by EnKF and ES at 6th time step

Factor of safety (FoS) estimation



Factor of safety estimation by EnKF and ES at 6th time step by true, prior and estimated parameters

Factor of safety (FoS) estimation



Factor of safety estimation by EnKF and ES at 6th time step by true, prior and estimated parameters

Conclusions

- Data assimilation provides a promising estimate of parameters and factor of safety.
- The use of a Mohr-Coulomb (MC) constitutive model in data assimilation does not always output a reliable factor of safety estimation.
- However, the hardening soil (HS) model is able to yields a reliable factor of safety estimation using data assimilation under different loading conditions (close-to-failure, far-from-failure).
- Ensemble Kalman filter (EnKF) provides better results than ensemble smoother (ES).

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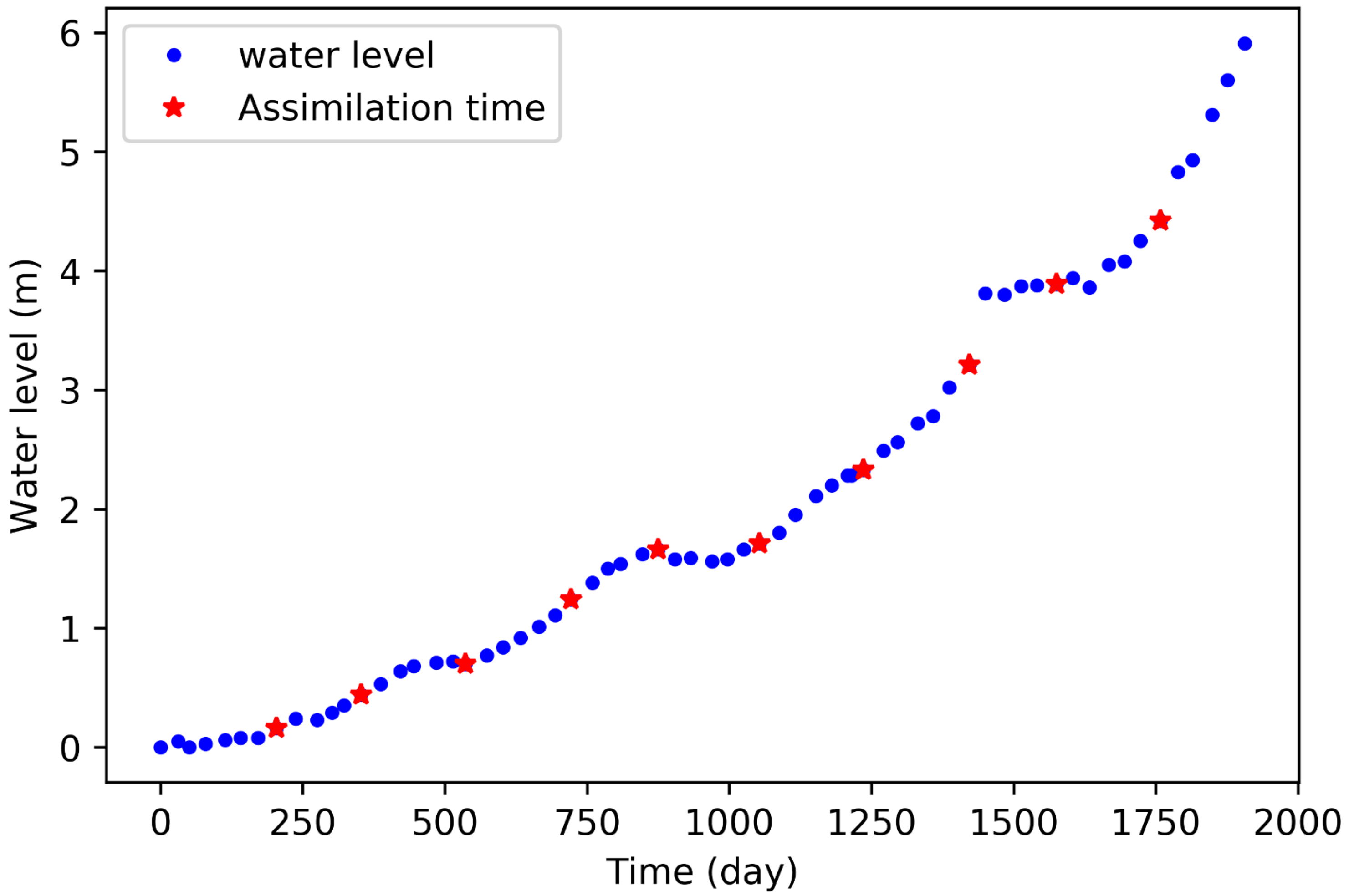
Thank you for listening!

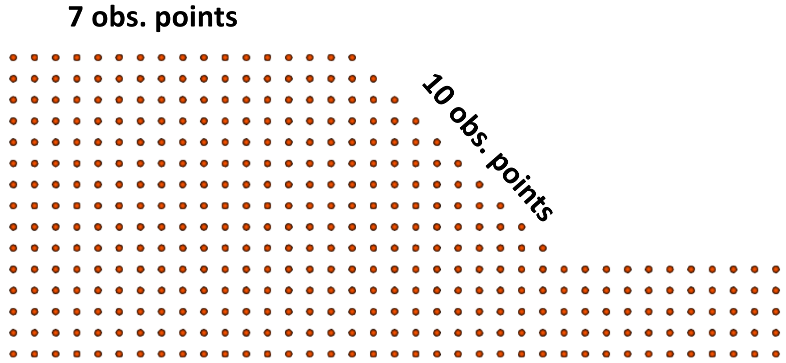
References

- Tandeo, P., Ailliot, P., Bocquet, M., Carrassi, A., Miyoshi, T., Pulido, M., & Zhen, Y. (2020). A review of innovation-based methods to jointly estimate model and observation error covariance matrices in ensemble data assimilation. *Monthly Weather Review*, *148*(10), 3973-3994.

Appendix

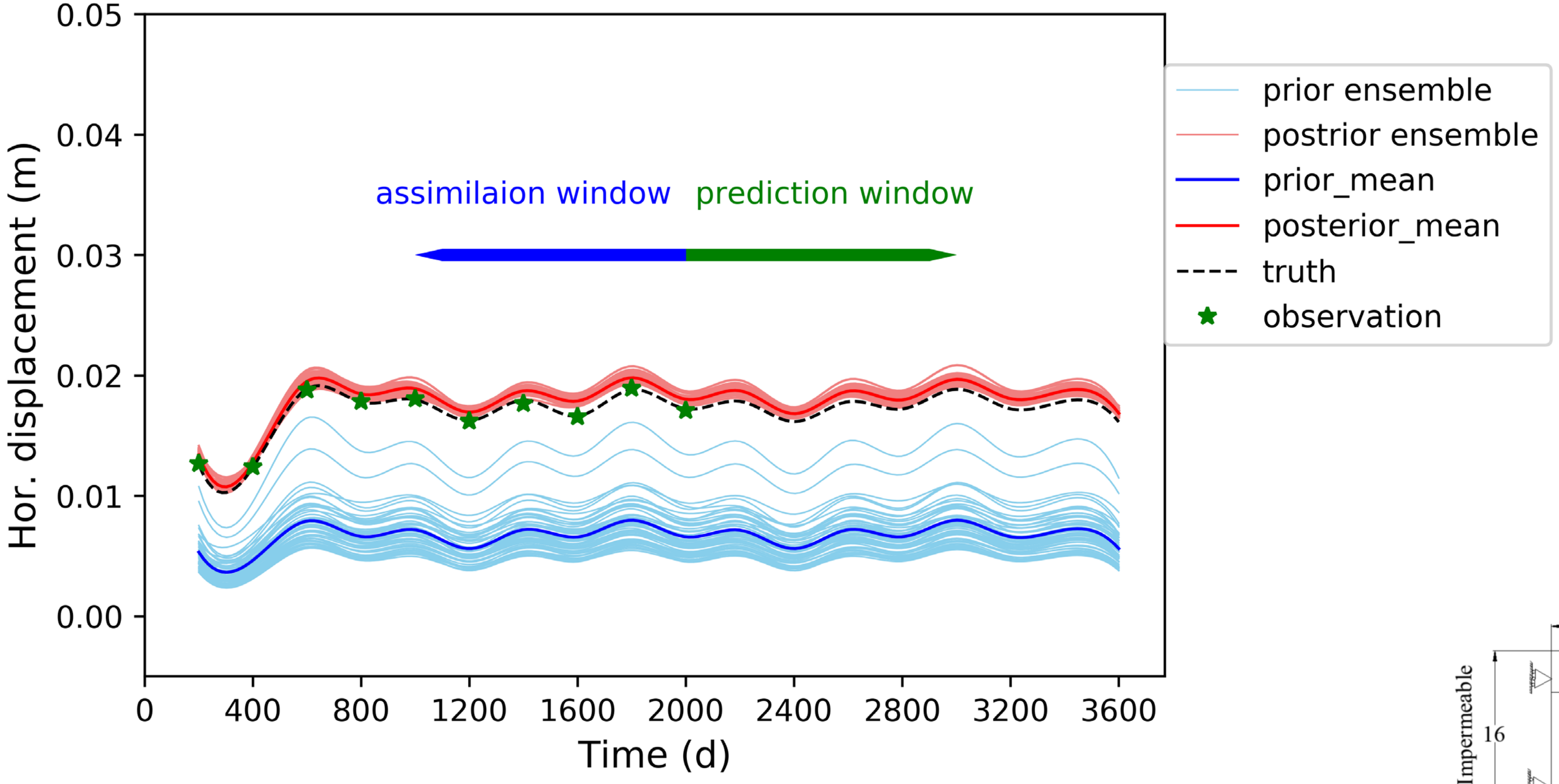
Synthetic problem



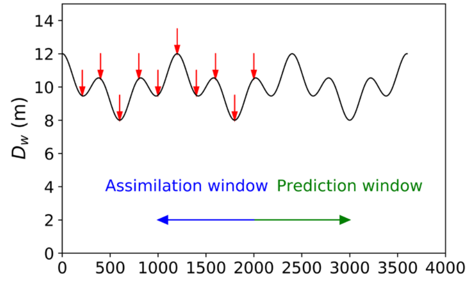


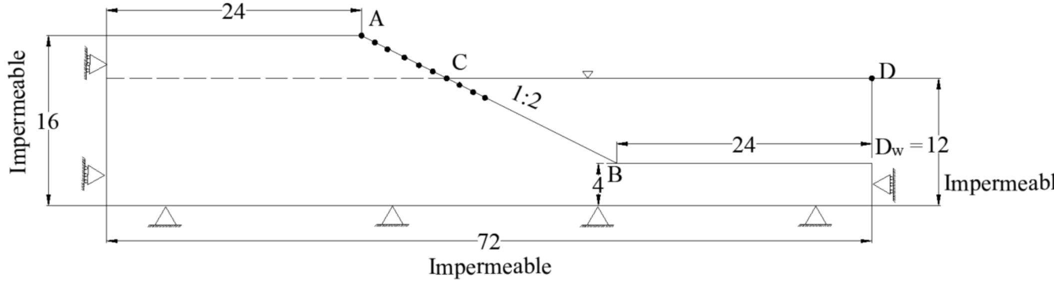
Slope geometry

Displacement estimation



Ensemble prediction of the horizontal displacement at point A of the slope

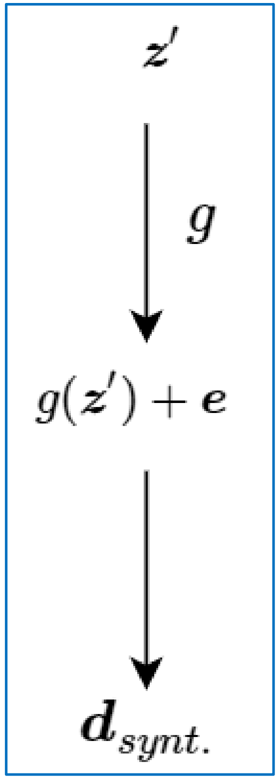




Twin experiment

1

1. Synthetic observations



$\boldsymbol{z}'$: synthetically created state-parameter vector

$\boldsymbol{e}$: normally distributed measurement noise

$\boldsymbol{d}_{synth}$: vector containing synthetic measurements

$\begin{bmatrix} \boldsymbol{z}_1^{\mathrm{f}} \\ \boldsymbol{z}_2^{\mathrm{f}} \\ \dots \\ \boldsymbol{z}_{N_e}^{\mathrm{f}} \end{bmatrix}$: ensemble of prior state-parameter vectors

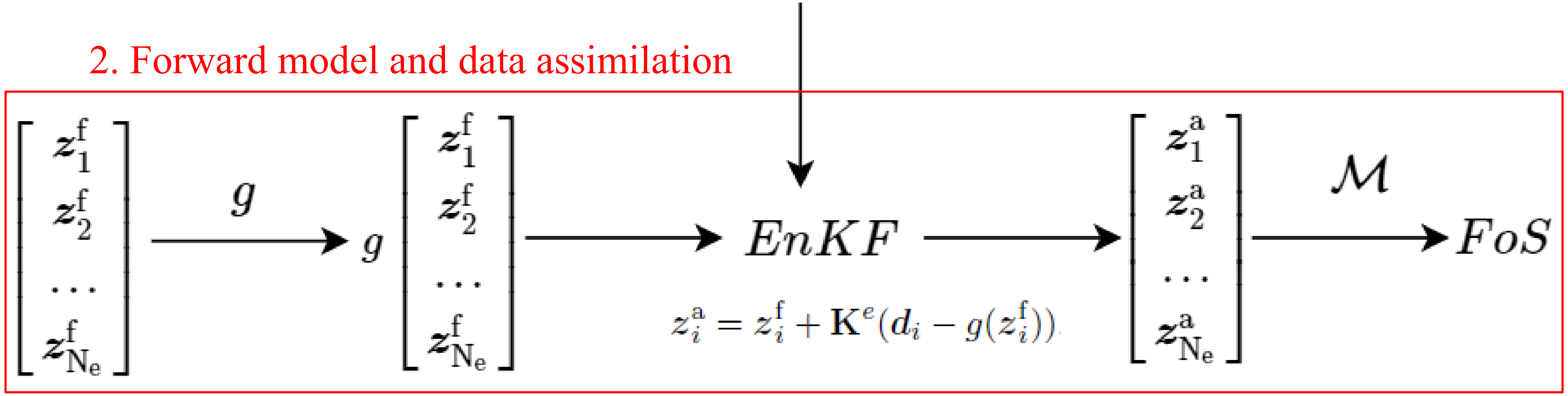
g: model operator

$EnKF$: data assimilation (recursive EnKF)

$\begin{bmatrix} \boldsymbol{z}_1^{\mathrm{a}} \\ \boldsymbol{z}_2^{\mathrm{a}} \\ \dots \\ \boldsymbol{z}_{N_e}^{\mathrm{a}} \end{bmatrix}$: ensemble of posterior state-parameter estimates

2

2. Forward model and data assimilation



Data Assimilation: Ensemble Kalman filter (EnKF)

$$\boldsymbol{y} = g(\boldsymbol{z}), \qquad \boldsymbol{z} = \begin{pmatrix} \boldsymbol{x} & \boldsymbol{\theta} \end{pmatrix}^{\mathrm{T}}$$

$\boldsymbol{y}$: model output vector at measurement location

g: state vector

$\boldsymbol{z}$: state-parameter vector

$\boldsymbol{x}$: state vector

$\boldsymbol{\theta}$: parameter vector

J: cost function

$\boldsymbol{d}$: measurement vector

Bayes's theorem:

$$f(\boldsymbol{z}, \boldsymbol{y}|\boldsymbol{d}) \propto f(\boldsymbol{d}|\boldsymbol{y}) f(\boldsymbol{y}|\boldsymbol{z}) f(\boldsymbol{z})$$

$$f(\boldsymbol{z}|\boldsymbol{d}) \propto \exp\{-\frac{1}{2} J\}$$

$$\Rightarrow J(\boldsymbol{z}) = (\boldsymbol{z} - \boldsymbol{z}^{\mathrm{f}})^{\mathrm{T}} \mathbf{C}_{\boldsymbol{zz}}^{-1} (\boldsymbol{z} - \boldsymbol{z}^{\mathrm{f}}) + (g(\boldsymbol{z}) - \boldsymbol{d})^{\mathrm{T}} \mathbf{C}_{\boldsymbol{dd}}^{-1} (g(\boldsymbol{z}) - \boldsymbol{d})$$

In EnKF:

$$\Rightarrow \boldsymbol{z}_i^{\mathrm{a}} = \boldsymbol{z}_i^{\mathrm{f}} + \mathbf{K}^e (\boldsymbol{d}_i - g(\boldsymbol{z}_i^{\mathrm{f}}))$$

$$\Rightarrow \mathbf{K}^e = \mathbf{C}_{\boldsymbol{zz}}^e \mathbf{G} (\mathbf{G} \mathbf{C}_{\boldsymbol{zz}}^e \mathbf{G}^{\mathrm{T}} - \mathbf{C}_{\boldsymbol{dd}})^{-1}$$

$$\boldsymbol{z} = \begin{pmatrix} \boldsymbol{x} & \boldsymbol{\theta} \end{pmatrix}^{\mathrm{T}}$$

$\boldsymbol{z}$: state-parameter vector

$\boldsymbol{x}$: state vector

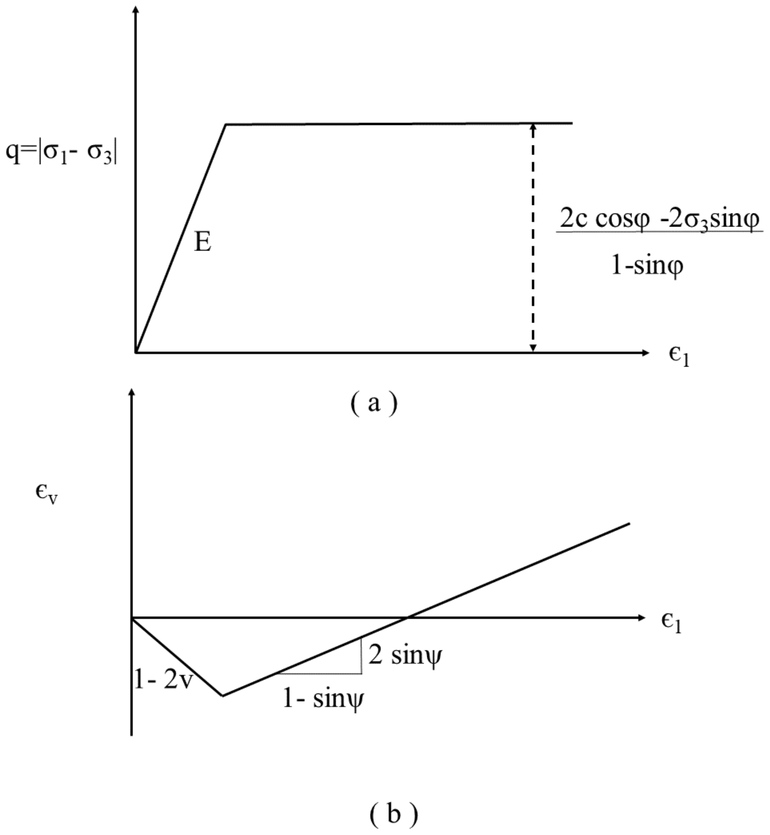
$\boldsymbol{\theta}$: parameter vector

$$\boldsymbol{z}_i^{\mathrm{a}} = \boldsymbol{z}_i^{\mathrm{f}} + \mathbf{K}^e (\boldsymbol{d}_i - g(\boldsymbol{z}_i^{\mathrm{f}}))$$

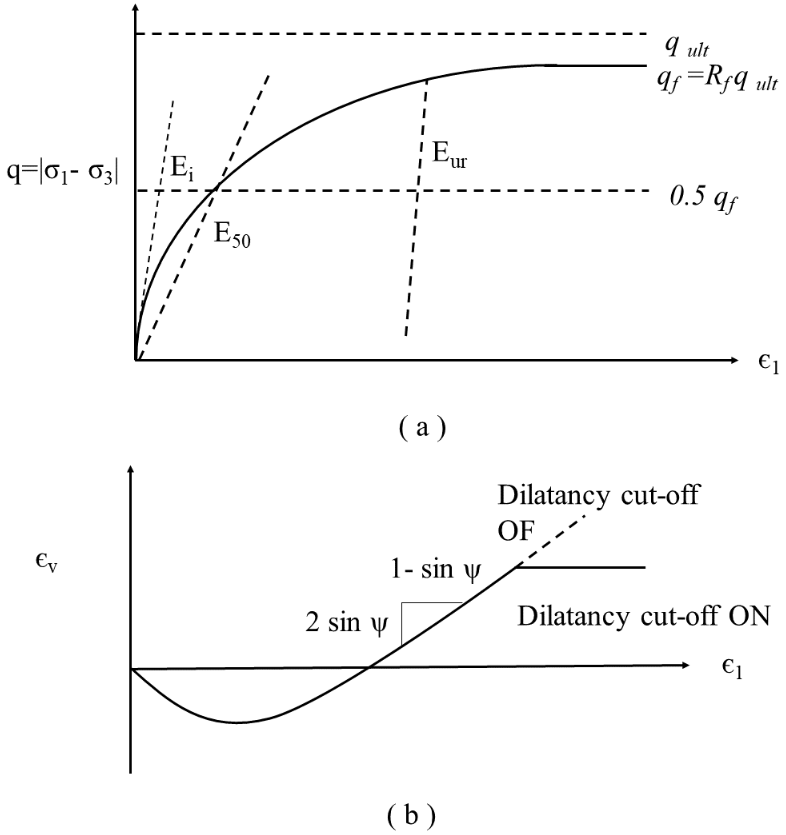
$$\mathbf{K}^e = \mathbf{C}_{\boldsymbol{zz}}^e \mathbf{G} (\mathbf{G} \mathbf{C}_{\boldsymbol{zz}}^e \mathbf{G}^{\mathrm{T}} - \mathbf{C}_{\boldsymbol{dd}})^{-1}$$

Constitutive models

A set of mathematical equations which represents stress-strain behaviour in a given material is called the constitutive model.



Mohr-Coulomb (MC) model



Hardening soil (HS) model

HS model

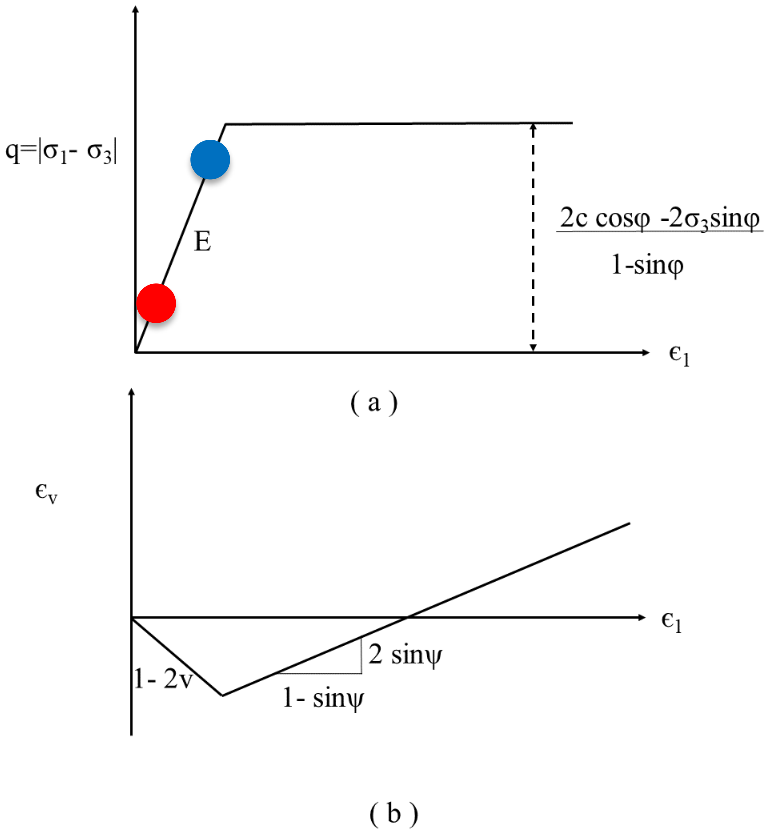
$$E_{ur} = E_{ur}^{ref}\left(\frac{c\cos(\phi)-\sigma_3\sin(\phi)}{c\cos(\phi)-p_{ref}\sin(\phi)}\right)^m$$

$$E_{50} = E_{50}^{ref}\left(\frac{c\cos(\phi)-\sigma_3\sin(\phi)}{c\cos(\phi)-p_{ref}\sin(\phi)}\right)^m$$

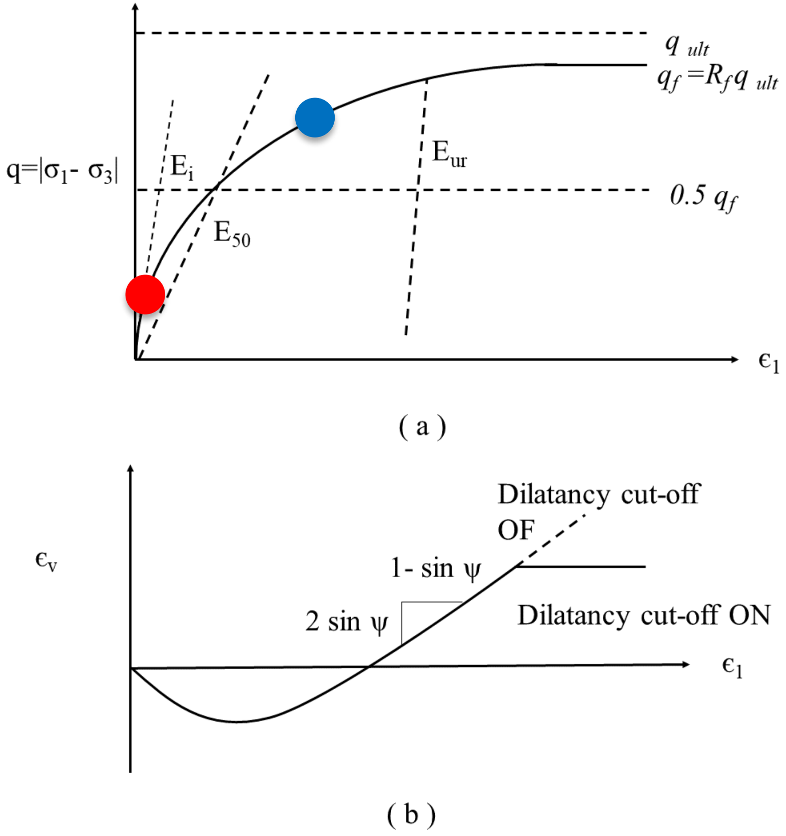
$$E_{oed} = E_{oed}^{ref}\left(\frac{c\cos(\phi)-\sigma_3\sin(\phi)}{c\cos(\phi)-p_{ref}\sin(\phi)}\right)^m$$

Constitutive models

A set of mathematical equations which represents stress-strain behaviour in a given material is called the constitutive model.



Mohr-Coulomb (MC) model



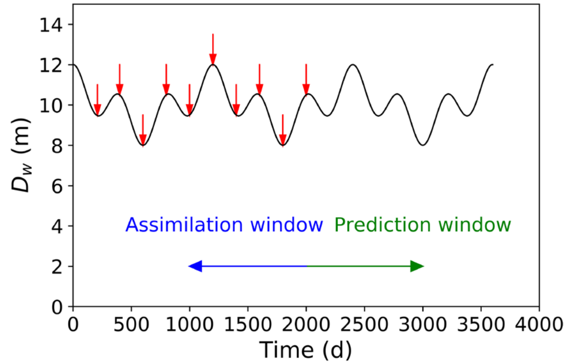
Hardening soil (HS) model

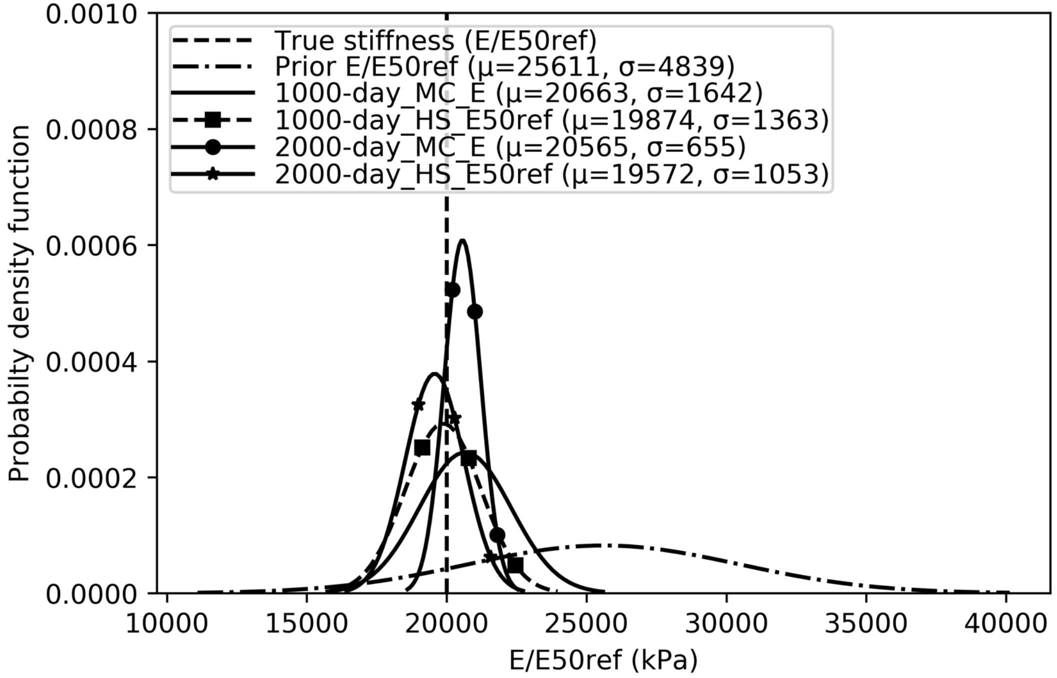
Close-to-failure

Far-from-failure

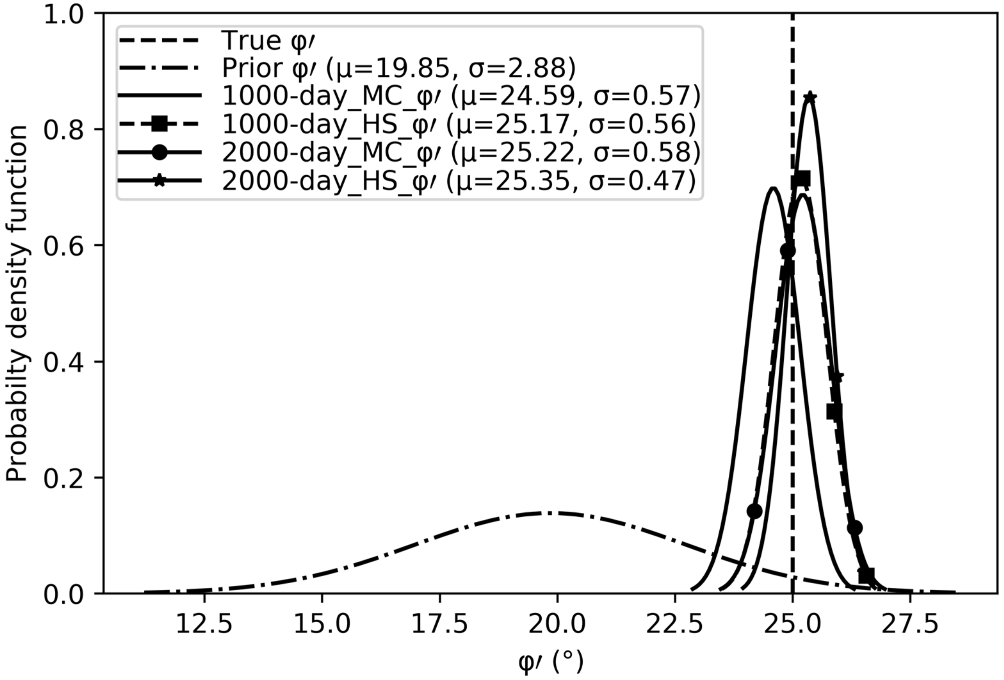
Results and Discussion

-Parameter (stiffness and strength) estimation for close-to-failure case





Estimation of stiffness for the MC and HS soil models at t=1000 and t=2000 days



Estimation of friction angle for the MC and HS soil models at t=1000 and t=2000 days

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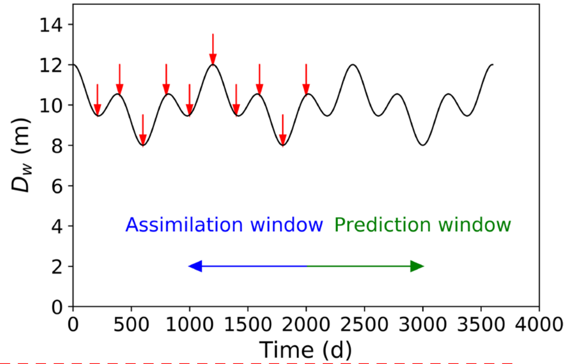
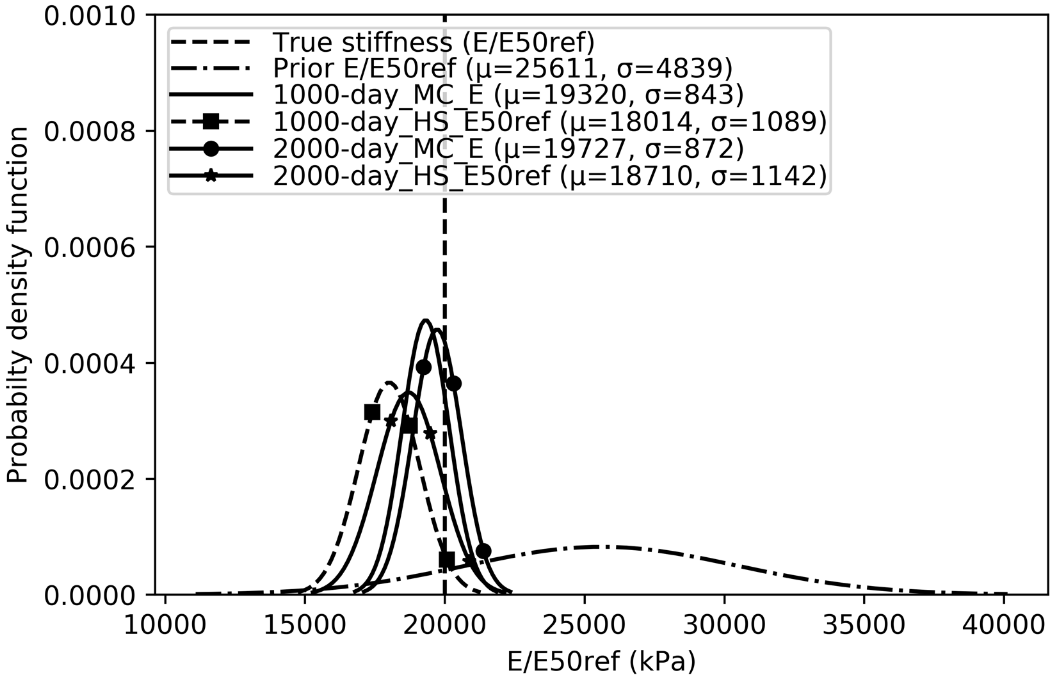
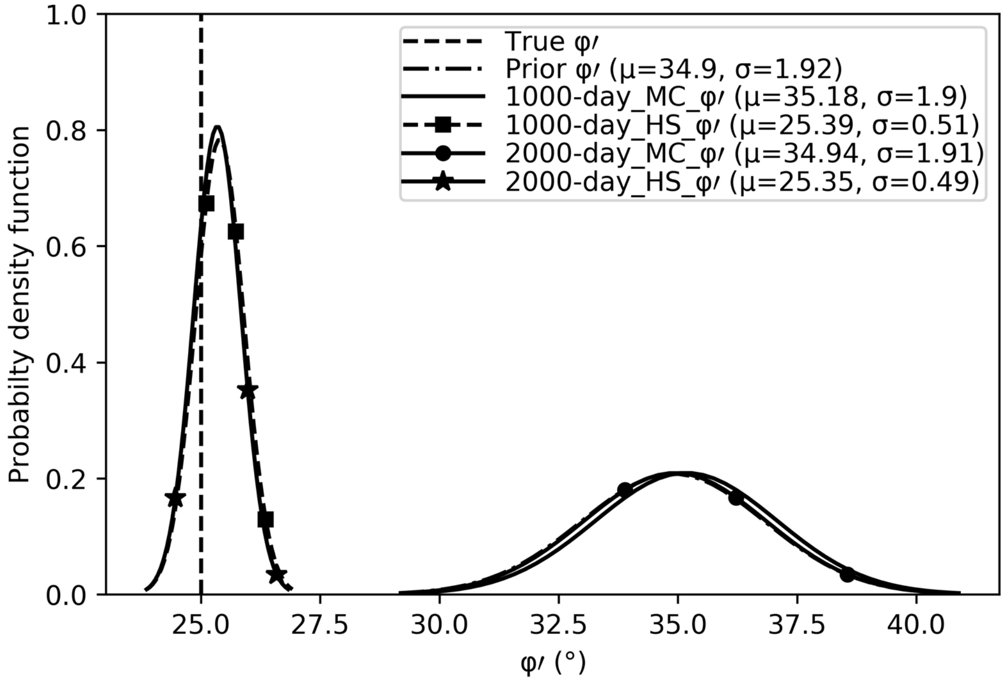
Results and discussion



-Parameter (stiffness and strength) estimation



Estimation of stiffness for the MC and HS soil models at t=1000 and t=2000 days



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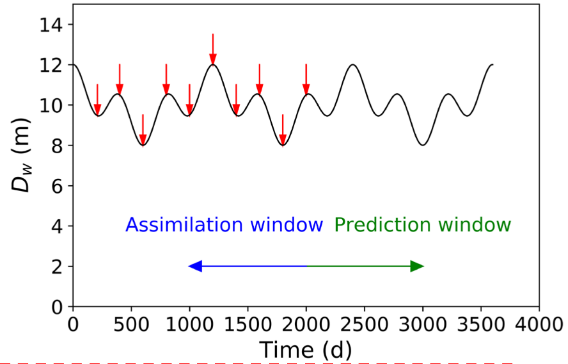
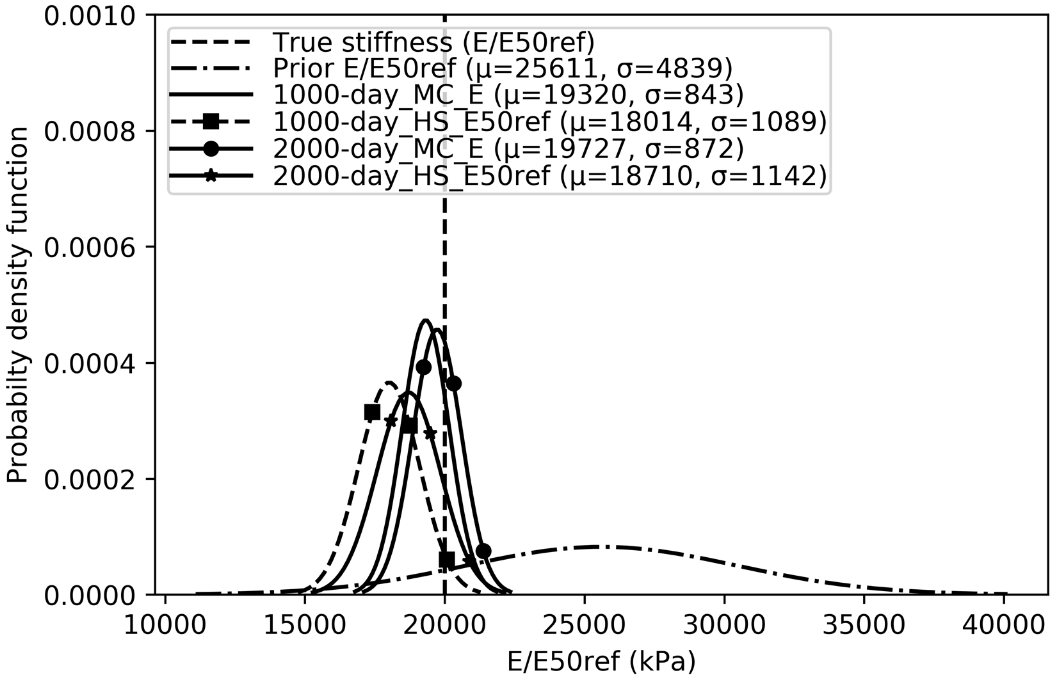
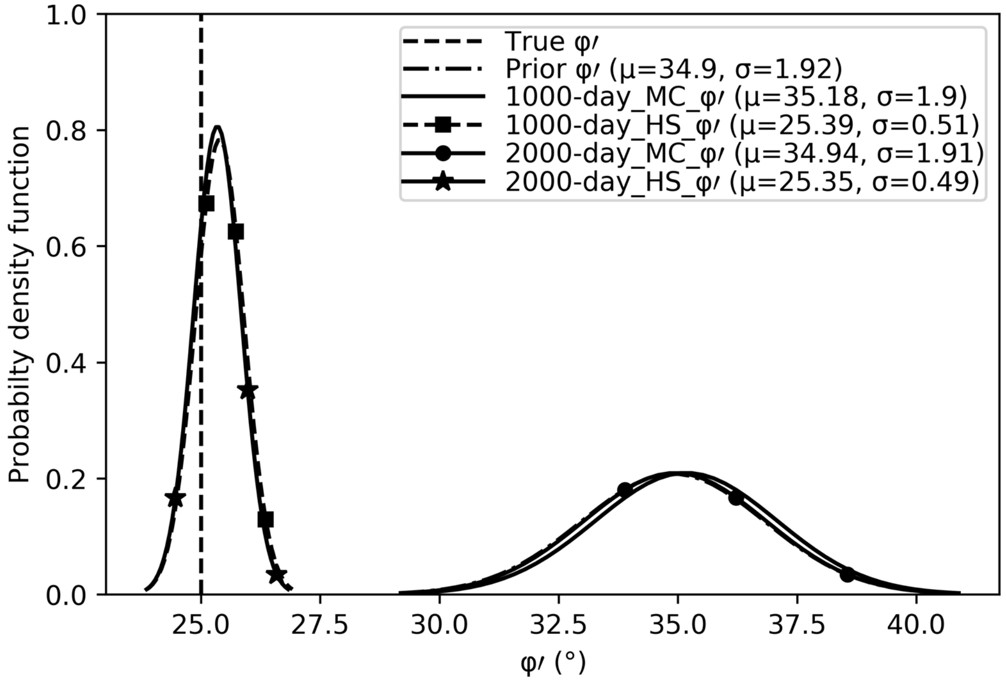


-Parameter (stiffness and strength) estimation



Estimation of stiffness for the MC and HS soil models at t=1000 and t=2000 days



Estimation of friction angle for the MC and HS soil models at t=1000 and t=2000 days

TUDelft